Final Examination

Math 3

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Name:____

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Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided. Take a moment now to print your name and section clearly on your Scantron form and on this page of your exam booklet. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form and your exam booklet. There are 30 multiple choice problems each worth 5 points. Check to see that you have 15 pages of questions plus the cover page.

1. For the function $f(x) = \dfrac{x-1}{x^2} = x^{-1} - x^{-2}$, its domain is

(a) $(-\infty, 1) \cup (1, \infty)$

(b) $(-\infty, 0) \cup (0, \infty)$

(c) $(-\infty, \infty)$

(d) $(0, \infty)$

(e) none of the above

2. For the same function $f(x)$ as in problem 1, its horizontal and vertical asymptotes are

(a) $y = 0, x = 0$

(b) $y = 1, x = 1$

(c) $y = 1, x = 0$

(d) $y = 0, x = 1$

(e) No choice above gives the complete and correct list of horizontal and vertical asymptotes.

3. Which item describes the graph of the function $f(x) = x^{-1} - x^{-2}$ from problems 1 and 2?

(a) increasing on $(-\infty, 0)$, decreasing on $(0, 2)$, and increasing on $(2, \infty)$; concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$.

(b) decreasing on $(-\infty, 0)$, increasing on $(0, 2)$, and decreasing on $(2, \infty)$; concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$.

(c) decreasing on $(-\infty, 0)$, increasing on $(0, 2)$, and decreasing on $(2, \infty)$; concave down on $(-\infty, 0)$, concave down on $(0, 3)$, and concave up on $(3, \infty)$.

(d) increasing on $(-\infty, 0)$, decreasing on $(0, 2)$, and increasing on $(2, \infty)$; concave up on $(-\infty, 0)$, concave down on $(0, 3)$, and concave up on $(3, \infty)$.

(e) none of the above

4. Suppose the graph of the curve $y = x^2$ is shifted to the left by 1/2 and then shifted down by 1/4. What is the equation for the shifted curve?

(a) $y = x^2 + x$

(b) $y = x^2 - \frac{3}{4}$

(c) $y = x^2 - x$

(d) $y = x^2 + x + \frac{1}{4}$

(e) $y = x^2 + x - \frac{1}{4}$

5. The inverse of the function $y = \dfrac{1}{x}$ is the function

(a) $y = \ln x$

(b) $y = x$

(c) $y = \dfrac{x+1}{x-1}$

(d) $y = \dfrac{1}{x}$

(e) none of the above

6. Find $\dfrac{dy}{dx}$ when $y = x\ln(x) - x$. It is

(a) $\ln(x)$

(b) $\dfrac{x}{\ln(x)} - 1$

(c) $\ln(x) - 1$

(d) $\dfrac{1}{1+x^2} - 1$

(e) none of the above

7. Find $\frac{dy}{dx}$ for $y = x^2 \sin(1/x)$. It is

(a) $-\cos(1/x)$

(b) $-\cos(1/x) + 2x\sin(1/x)$

(c) $2x\sin(1/x)$

(d) $2x - x^{-2}\cos(1/x)$

(e) none of the above

8. Find the slope of the tangent line to the curve $y = (\arctan(3x))^3$ at the point $\left(\frac{1}{3}, \frac{\pi^3}{64}\right)$.

(a) $\frac{9\pi^2}{32}$

(b) $\frac{3\pi^2}{32}$

(c) $\frac{3\pi^2}{16}$

(d) $\frac{9}{2}$

(e) none of the above

9. Find the derivative of $f(x) = \dfrac{\sqrt{x}-1}{\sqrt{x}+1}$.

(a) $\dfrac{1}{(\sqrt{x}+1)^2}$

(b) $\dfrac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(c) $-\dfrac{\sqrt{x}}{(\sqrt{x}+1)^2}$

(d) $\dfrac{1}{x+\sqrt{x}}$

(e) 1

10. The limit $\lim_{x\to 0} \dfrac{x+|x|}{2x}$ is

(a) 0

(b) 1

(c) 2

(d) ∞

(e) does not exist

11. Which of the following is true for the function

$$f(x) = \begin{cases} -x, & x \leq 0 \\ 1/x, & 0 < x < 3 \\ x/9, & 3 \leq x. \end{cases}$$

(a) $f(x)$ is continuous at $x = 0$ and $x = 3$

(b) $f(x)$ is undefined at $x = 0$ and continuous at $x = 3$

(c) $f(x)$ is discontinuous at both $x = 0$ and $x = 3$

(d) $f(x)$ is discontinuous at $x = 0$ and continuous at $x = 3$

(e) none of the above

12. The limit $\lim\limits_{h \to 0} \dfrac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$ is

(a) 0

(b) 2

(c) $\sqrt{2}$

(d) $\dfrac{1}{2}$

(e) none of the above

13. This problem involves finding the absolute maximum and absolute minimum of the function $f(x) = x^3 - x + 2$ restricted to the closed interval $[0, 2]$. Which of the following statements is correct?

(a) $f(x)$ has both an absolute maximum and absolute minimum at the end points.

(b) $f(x)$ has both an absolute maximum and absolute minimum at interior points.

(c) $f(x)$ has an absolute maximum at an end point and an absolute minimum at an interior point.

(d) $f(x)$ has an absolute maximum at an interior point and an absolute minimum at an end point.

(e) none of the above

14. Consider the functions

$$\begin{aligned} f(x) &= 5 + 10x - x^2, \\ g(x) &= x^3 - 3x^2 + 3x, \\ h(x) &= x^4 + 4x^3 + 6x^2 + 4x. \end{aligned}$$

Which statement is completely true?

(a) $f(x)$ and $g(x)$ have absolute maximum points, $h(x)$ has an absolute minimum point.

(b) $f(x)$ and $h(x)$ have absolute maximum points, $g(x)$ has a local minimum point.

(c) $f(x)$ and $h(x)$ have absolute minimum points, $g(x)$ has a local minimum point.

(d) $f(x)$ has an absolute maximum point, $g(x)$ has both a local minimum point and a local maximum point, $h(x)$ has an absolute minimum point.

(e) None of the above is completely true.

15. Let $f(x)$ be differentiable on $(-\infty, \infty)$, and suppose that $f(-1) = 5$, $f(2) = -9$, and $f(7) = 3$. Which of the following can be deduced from the given information?

(a) $f'(x) > 0$ for all x in the interval $(-1, 7)$

(b) $f(x)$ has a local minimum at $x = 2$

(c) f has at least 2 roots

(d) f has at least 3 roots

(e) None of the above can be deduced from the given information.

16. Suppose you use Newton's method to find a value of x where $x^3 + x = 1$ starting with an initial guess of $x_0 = 0$. Call the next value x_1 and the one after that x_2. Then x_2 is

(a) 1

(b) 0.75

(c) 0.6860465

(d) 0.6823278

(e) none of the above

17. If $\dfrac{dy}{dx} = \sin(2x)$ and $y(0) = 0$, find $y(\pi/4)$. It is

(a) $-\dfrac{1}{2}$

(b) $\dfrac{1}{2}$

(c) $-\dfrac{\sqrt{2}}{2}$

(d) $\dfrac{\sqrt{2}}{2}$

(e) none of the above

18. Which differential equation below does *not* lend itself to the method of separation of variables?

(a) $\dfrac{dy}{dx} = xy$

(b) $\dfrac{dy}{dx} = xy + x$

(c) $\dfrac{dy}{dx} = x + y$

(d) $\dfrac{dy}{dx} = x/y$

(e) none of the above

19. Find a solution $y = y(x)$ to the equation $\dfrac{dy}{dx} = y + \dfrac{1}{y}$ and $y(0) = 1$. It is

(a) 1

(b) $\frac{1}{2}y^2 + \ln y$

(c) $2e^x - 1$

(d) $\sqrt{2e^{2x} - 1}$

(e) none of the above

20. During the first few days, a certain cell culture grew at a rate proportional to the number of cells present. After one day, the population was 5000 cells and after 3 days it was 15000 cells. Approximately how many cells were present after 2 days? (Note that $e \approx 2.72, \pi \approx 3.14, \sqrt{2} \approx 1.41, \sqrt{3} \approx 1.73, \sqrt{5} \approx 2.24, \ln 2 \approx 0.69, \ln 3 \approx 1.10$.)

(a) 7500

(b) 8500

(c) 10000

(d) 11500

(e) 13000

21. Compute $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^3$. It is

(a) 0.25

(b) 3.75

(c) 4

(d) 4/3

(e) none of the above

22. Compute $\int (x^2+1)^2\,dx$. It is

(a) $\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

(b) $\frac{1}{3}(x^2+1)^3 + C$

(c) $\frac{1}{3}(x^2+1)^3/(2x) + C$

(d) $\frac{1}{3}(\arctan(x))^3 + C$

(e) none of the above

23. If you were to try and compute $\int x^2(3x^3 - 5)^6 \, dx$ by the method of substitution, then a good choice would be

(a) $u = x^2$

(b) $u = x^6$

(c) $u = 3x^3 - 5$

(d) $u = x^2(3x^3 - 5)^6$

(e) none of the above

24. $\int \frac{1}{\sqrt{1 - 25x^2}} \, dx$ is

(a) $5 \arcsin(5x) + C$

(b) $\arcsin(25x) + C$

(c) $\frac{1}{5} \arcsin(5x) + C$

(d) $\frac{1}{5} \arccos(5x) + C$

(e) $\frac{1}{5} \arctan(5x) + C$

25. Let $F(x) = \int_1^x \ln t \, dt$ so that $F(x)$ is defined for all $x > 0$. Which statement is true?

(a) $F(x)$ has an absolute minimum at $x = 1$.

(b) $F(x)$ is increasing on the interval $(0, \infty)$.

(c) $F(x)$ is decreasing on the interval $(0, \infty)$.

(d) $F(x)$ has an absolute maximum at $x = 1$.

(e) None of the above statements is true.

26. The length of the curve $y = e^x + \frac{1}{4}e^{-x}$ on the interval $0 \leq x \leq \ln 2$ is

(a) $\frac{17}{8}$

(b) $\frac{9}{8}$

(c) $\frac{9}{4}$

(d) $e^2 + \frac{1}{4}e^{-2}$

(e) 2

27. The Trapezoidal Rule approximation T_4 for $\int_1^3 \frac{1}{x^3+2}\,dx$ is

(a) $\frac{1}{4}\left(\frac{1}{3}+\frac{1}{(1.5)^3+2}+\frac{1}{10}+\frac{1}{(2.5)^3+2}+\frac{1}{29}\right)$

(b) $\frac{1}{2}\left(\frac{1}{3}+\frac{2}{(1.5)^3+2}+\frac{2}{10}+\frac{2}{(2.5)^3+2}+\frac{1}{29}\right)$

(c) $\frac{1}{2}\left(\frac{1}{3}+\frac{2}{3(1.5)^2}+\frac{2}{12}+\frac{2}{3(2.5)^2}+\frac{1}{29}\right)$

(d) $\frac{1}{4}\left(\frac{1}{3}+\frac{2}{(1.5)^3+2}+\frac{2}{10}+\frac{2}{(2.5)^3+2}+\frac{1}{29}\right)$

(e) none of the above

28. Which integral is the area of the region in the first quadrant bounded by the curves $y=\frac{x^2}{8}$ and $y=\sqrt{x}$?

(a) $\int_0^4 \sqrt{x}-\frac{x^2}{8}\,dx$

(b) $\int_0^4 \frac{x^2}{8}-\sqrt{x}\,dx$

(c) $\int_0^8 \sqrt{x}-\frac{x^2}{8}\,dx$

(d) $\int_0^2 \frac{x^2}{8}-\sqrt{x}\,dx$

(e) $\int_0^4 x-\frac{x^2}{8}\,dx$

29. Which of the following gives the area enclosed by the unit circle?

(a) $2\int_0^1 \sqrt{1-x^2}\,dx$

(b) $4\int_0^1 \sqrt{1-x^2}\,dx$

(c) $\int_{-1}^1 \sqrt{1-x^2} + \int_{-1}^1 -\sqrt{1-x^2}\,dx$

(d) $2\int_{-1}^1 -\sqrt{1-x^2}\,dx$

(e) none of the above

30. Find the centroid of the region composed of the rectangles

$$R_1 : -2 \le x \le -1, \quad 0 \le y \le 2$$
$$R_2 : -1 \le x \le 1, \quad 0 \le y \le 1$$
$$R_3 : 1 \le x \le 3, \quad 0 \le y \le 3$$

(a) $(15, 12)$

(b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(\frac{9}{10}, \frac{6}{5}\right)$

(d) $\left(\frac{6}{5}, \frac{9}{10}\right)$

(e) $\left(\frac{1}{2}, \frac{1}{2}\right)$