Math 3, Fall 2005
November 28, 2005

The Final Exam this year will be on Saturday, December 3 from 11:30 a.m. to 2:30 p.m. in Spaulding (in the Hopkins Center). The exam is completely multiple choice. Below are the problems from the final exam of Math 3, Fall 2000 when two hours were allowed for the exam. This year's exam has a similar format but must be completed in three hours. There will be 30 multiple choice problems each worth 5 points, for a total of 150 points. The exam is cumulative, but a disproportionate amount will be on material studied since the second hour exam. As in previous exams, you are not allowed to use calculators, books, or notes of any kind. We will be using this exam as the basis of our review in class on 11/28 and 11/30. --Professors Lahr, Elizalde, and Ionescu

1. The derivative of the function $y = \frac{x^2+1}{5}$ is:

   A. $\frac{10x - \left(x^2+1\right)}{25}$ B. $\frac{2}{5}x$ C. $\frac{2}{5}x + \frac{1}{5}$

   D. $\frac{1}{15}x^3 + \frac{1}{5}x$ E. none of these

2. The limit $\lim\limits_{x\to 2} \frac{x+1}{x^2 - x - 2}$ equals:

   A. 0 B. ∞ C. $-\infty$ D. does not exist E. none of these

3. If $f(x) = \frac{x-1}{x+1}$, then the limit $\lim\limits_{x\to\infty} f'(x)$ of the <u>derivative</u> of f as x approaches infinity equals:

   A. 0 B. 1 C. 2 D. does not exist E. none of these

4. If $f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ \frac{24}{\pi}\arctan(x) & \text{if } x \le 1 \end{cases}$, then the one-sided limit $\lim\limits_{x\to 1^+} f(x)$ equals:

   A. 2 B. 4 C. 6 D. does not exist E. none of these

5. The inverse of the function $f(x) = 5^x$ is $f^{-1}(x) =$:

   A. $5^{1/x}$ B. $\frac{\ln x}{\ln 5}$ C. $\frac{1}{5^x}$ D. $\ln(5x)$ E. none of these

6. The function $f(x)=\begin{cases}\sin\left(x+\frac{\pi}{4}\right) & \text{if } x<0\\ \frac{1}{\sqrt{x+c}} & \text{if } x\geq 0\end{cases}$, is continuous at $x=0$ if:

A. $c=0$ B. $c=1$ C. $c=2$

D. f is not continuous at 0 no matter what c equals

E. none of these

7. The limit $\lim_{h\to 0}\frac{\ln(e+h)-1}{h}$ equals:

A. 0 B. $\frac{1}{e}$ C. 1 D. does not exist E. none of these

8. The slope of the line tangent to the graph of $f(x)=x^3e^{2x}$ at the point $(1,e^2)$ is:

A. e^2 B. $2e^2$ C. $3e^2$ D. $4e^2$ E. none of these

9. The slope of the line tangent to the curve $x\sqrt{y}+2=y$ at the point (-1, 1) is:

A. 1 B. 1/2 C. 2/3 D. does not exist E. none of these

10. The derivative of the function $y=(1+x)^2+2^{1+x}$ is:

A. $2(1+x)+2^{1+x}\ln 2$ B. $2(1+x)+(1+x)2^x$

C. $(1+x)^2\ln(1+x)+(1+x)2^x$

D. $2(1+x)+2^{1+x}\ln(1+x)$ E. none of these

11. If $f(x)=\sqrt{\ln(\ln x)}$, then $f'(e^e)$ equals:

A. $\frac{1}{2}$ B. $\frac{1}{2e}$ C. $\frac{1}{2e^{e+1}}$ D. $\frac{2}{3}$ E. none of these

12. How many points of inflection does the graph of $f(x)=x^5-5x^4$ have?

A. none B. one C. two D. three E. four

13. The value of the integral $\int_0^1\left(3x^2+\sec^2 x\right)dx$ is:

A. $1+\tan 1$ B. $3+\tan 1$ C. $1+\frac{1}{3}\sec^3 1$

D. $3+\frac{1}{3}\sec^3 1$ E. none of these

14. The integral $\int_0^{\ln 2} x e^x \, dx$ equals:

A. 1 B. 2 C. $2\ln 2-2$ D. $2\ln 2-1$ E. none of these

15. The integral $\int_0^{1/4} \frac{1}{\sqrt{1-(2x)^2}} \, dx$ equals:

A. $\frac{\pi}{4}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{8}$ D. $\frac{\pi}{12}$ E. none of these

16. If y is the solution of the IVP $\frac{dy}{dx}=x^2 y$, $y(0)=2$, then $y\left(3^{1/3}\right)$ equals:

A. 0 B. e C. $2e$ D. $3e$ E. none of these

17. If $F(x)=\int_{x^2}^{1} \sqrt{9+t^2}\, dt$, then $F'(2)$ equals:

A. -20 B. -5 C. 0 D. 5 E. none of these

18. The area of the region bounded by the y-axis and the curves $y=\sin x$, $0 \le x \le \frac{\pi}{4}$, and $y=\cos x$, $0 \le x \le \frac{\pi}{4}$ is:

A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\sqrt{2}-1$ D. $\sqrt{2}+1$ E. none of these

19. The region bounded by the curve $y=e^x$, $x=0$, $y=0$, and $x=\frac{1}{2}\ln 5$ is rotated about the x-axis. The volume of the solid generated is: [OMIT Volumes of Solids of Revolution]

A. π B. 2π C. 3π D. 4π E. none of these

20. The approximation to $\sqrt{49.1}$ given by the linearization of a suitable function at a suitable point is:

A. $7+\frac{1}{140}$ B. $7+\frac{1}{120}$ C. $7+\frac{1}{110}$ D. $7+\frac{1}{100}$ E. none of these

21 Newton's method applied once to approximate the solution of $x^3=11-2x$ starting with $x_0=2$ gives x_1 equal to:

A. $2+\frac{1}{2}$ B. $2-\frac{1}{2}$ C. $2+\frac{1}{14}$ D. $2-\frac{1}{14}$ E. none of these

22. The approximate value of the integral $\int_1^{5/2}\frac{1}{x}\,dx$ given by the trapezoid rule using three trapezoids is:

A. $\frac{1}{4}\left(1+\frac{4}{3}+1+\frac{2}{5}\right)=\frac{1}{2}\left(\frac{1}{2}+\frac{2}{3}+\frac{1}{2}+\frac{1}{5}\right)$

B. $\frac{1}{4}\left(2+\frac{4}{3}+1+\frac{2}{5}\right)=\frac{1}{2}\left(1+\frac{2}{3}+\frac{1}{2}+\frac{1}{5}\right)$

C. $\frac{1}{4}\left(2+\frac{4}{3}+1+\frac{4}{5}\right)=\frac{1}{2}\left(1+\frac{2}{3}+\frac{1}{2}+\frac{2}{5}\right)$

D. $\frac{1}{4}\left(\frac{4}{3}+1+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{2}{3}+\frac{1}{2}+\frac{2}{5}\right)$

E. none of these

23. The length of the curve $x^3+y^3=1$ from $x=2$ to $x=3$ is:

A. $\int_2^3\sqrt{1+\frac{x^2}{(1-x^3)^{2/3}}}\,dx$ B. $\int_2^3\sqrt{1+\frac{3x^2}{(1-x^3)^{2/3}}}\,dx$

C. $\int_2^3\sqrt{1+\frac{9x^4}{(1-x^3)^{4/3}}}\,dx$

D. $\int_2^3\sqrt{1+\frac{x^4}{(1-x^3)^{4/3}}}\,dx$ E. none of these

24. The integral $\int_0^1 x^2\,dx$ equals:

A. $\lim_{n\to\infty}\sum_{i=0}^{n-1}\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right)$ B. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$ C. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)^2\left(\frac{1}{n}\right)$

D. $\lim_{n\to\infty}\sum_{i=0}^{n-1}\left(\frac{2i}{n}\right)^2\left(\frac{1}{n}\right)$ E. none of these

25. The linearization of the function $f(x)=5+\int_8^x\frac{\sqrt[3]{t}}{t^2+1}\,dt$ at $x=8$ is:

A. $5+\frac{\sqrt[3]{x}}{x^2+1}(x-8)$ B. $5(x-8)$ C. $5+\frac{2}{65}(x-8)$

D. $5-\frac{2}{65}(x-8)$ E. none of these