Hour Exam 1

Math 3

Oct. 23, 2002

Name:_____

Instructor (circle): Lahr (8:45) Orellana (11:15)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All your answers to the multiple choice questions must be marked on the Scantron form provided, and your responses to the remaining questions must be written in this exam booklet. Take a moment now to print your name and section clearly on your Scantron form, and on your exam booklet. With regard to the multiple choice questions, you may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 10 multiple choice problems each worth 6 points, and there are 3 additional problems totaling 40 points. Check to see that you have 8 pages of questions plus this cover page.

Non-multliple choice questions:

Problem	Points	Score
1	15	
2	10	
3	15	
Total	40	

1. Suppose an object is falling near the surface of the earth. If you know that its acceleration is constant, what can you say about the distance function:

(a) The distance function is a linear function.

(b) The distance function is also constant.

(c) The distance function is quadratic.

(d) There is nothing that can be said without more information.

2. The limit

$$\lim_{x \to 0} \frac{x^2 - 2x + 4}{x^3 + 1}$$

is equal to

(a) 0

(b) 1

(c) ∞

(d) 4

(e) the limit does not exist.

3. The graph of $f(x) = -2^{-x}$ can be obtained from the graph of 2^x by

(a) Reflecting the graph of 2^x about the x-axis.

(b) Reflecting the graph of 2^x about the y-axis.

(c) Both reflecting the graph of 2^x about the x-axis and the y-axis.

(d) Reflecting in the origin.

(e) Both (c) and (d).

4. The domain and range of $f(x) = \sqrt{4 - x^2}$ are:

(a) Domain: $[0, \infty)$, Range: $[0, \infty)$

(b) Domain: $[4, \infty)$, Range: $[-2, 2]$

(c) Domain: $[-2, 2]$, Range: $(-\infty, \infty)$

(d) Domain: $[-2, 2]$, Range: $[0, \infty)$.

(e) None of the above.

5. Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x^2}$. What is the domain of $(g \circ f)(x) = g(f(x))$?

(a) All the real numbers.

(b) $[0, \infty)$

(c) $(0, \infty)$

(d) $(-\infty, 0) \cup (0, \infty)$

(e) None of the above.

6. The function $f(x) = \sin(x^2) + 3x^4 + 1$ is

(a) even and has an inverse function on its entire domain.

(b) odd and has an inverse function on its entire domain.

(c) neither odd nor even.

(d) odd and doesn't have an inverse function on its entire domain.

(e) None of the above.

7. The limit

$$\lim_{x\to\infty} \frac{2x^3+5x^2-3}{3x^4+2x+1}$$

is equal to:

(a) 0

(b) ∞

(c) Does not exist.

(d) 2/3

(e) None of the above.

8. For the function

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x+4 & 1 < x < 2 \end{cases}$$

(a) $f(1)$ exists.

(b) $\lim_{x\to 1} f(x)$ exists.

(c) $f(x)$ is continuous at $x = 1$.

(d) $x = 1$ is a removable discontinuity.

(e) (a), (b) and (d) are true.

9. The derivative of $f(x) = \sqrt{1 + \cos(x^2)}$ is:

(a) $\frac{1}{2\sqrt{1+\cos(x^2)}}$

(b) $\frac{\sin(x^2)}{2\sqrt{1+\cos(x^2)}}$

(c) $\frac{-x\sin(x^2)}{\sqrt{1+\cos(x^2)}}$

(d) $\frac{\sin(x^2)}{\sqrt{1+\cos(x^2)}}$

(e) None of the above.

10. The derivative of $f(x) = \frac{\sin(x)}{x}$ is:

(a) $\cos(x)/x$

(b) $\cos(x)$

(c) $-\cos(x)$

(d) $(x\cos(x) - \sin(x))/x^2$

(e) None of the above.

NON-MULTIPLE CHOICE. PLEASE SHOW ALL YOUR WORK.

1. (a) (5 pts) If

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x \leq 4 \end{cases}$$

determine if $\lim_{x \to 4} f(x)$ exists.

(b) (5 pts) Using the definition of continuity explain whether the function is continuous at $x = 4$.

(c) (5pts) Does the function have a unique tangent line at $x = 4$? Explain your reasoning (no credit will be given without justification).

2. (a) (5 pts) Show that the function f has a removable discontinuity at $x = -2$:

$$f(x) = \begin{cases} \frac{x^2-2x-8}{x+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$$

Explain using the definition of removable discontinuity.

(b) (5 pts) How should f be redefined at $x = -2$ so as to make it continuous there?

3. (a) (5 pts) Let $y = f(x)$ be a given function of x. State the limit definition of the derivative function $f'(x)$.

$f'(x) =$

(b) (5 pts) Use the limit definition of the derivative function to find the derivative $f'(x)$ of

$$f(x) = \sqrt{1 + 2x}$$

No credit will be given if the definition is not used.

(c) (5 pts) Find an equation of the tangent line at $x = 2$.