10/12/07 & 10/7/05
Barnett.

MATH 23 WORKSHEET : Wronskians

A) Compute Wronskian $W(t)$ for i) $t\sin t$, $\sin t$

ii) t^3 , $5t^3$

B) Show $\sinh t$, $\cosh t$ form a fundamental set of solutions* to $y'' - y = 0$

*Means: they solve the ODE (check!) & Wronskian doesn't vanish.

Hints:
$\sinh x := \frac{e^x - e^{-x}}{2}$
$\cosh x := \frac{e^x + e^{-x}}{2}$

Show these two solutions match $\begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$ and $\begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$

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~ SOLUTION ~

A) Compute Wronskian $W(t)$ for i) $\overbrace{t\sin t}^{y_1}$, $\overbrace{\sin t}^{y_2}$

$$W = y_1 y_2' - y_2 y_1' = \begin{vmatrix} t\sin t & \sin t \\ \sin t + t\cos t & \cos t \end{vmatrix} = \cancel{t\sin t\cos t} - \sin^2 t - \cancel{t\cos t\sin t}$$
$$= -\sin^2 t.$$

ii) t^3 , $5t^3$

$$W = \begin{vmatrix} t^3 & 5t^3 \\ 3t^2 & 15t^2 \end{vmatrix} = 15t^5 - 3(5)t^5 = 0 \text{ for all } t.$$

Note $y_1 - \frac{1}{5}y_2 = 0$. for all t (lin. dep.)

B) Show $\sinh t$, $\cosh t$ form a <u>fundamental set of solutions*</u> to $y'' - y = 0$

*Means: they solve the ODE (check!) & Wronskian doesn't vanish.

Hint: $\sinh x := \frac{e^x - e^{-x}}{2}$ $\cosh x := \frac{e^x + e^{-x}}{2}$

know e^t, e^{-t} are solns, from last Worksheet.

So $\sinh t$, $\cosh t$ are <u>linear combinations</u> of these $\Rightarrow$ also solutions.

$$W[\sinh t, \cosh t] = \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) - \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^x - e^{-x}}{2}\right) = -\frac{1}{2} - \frac{1}{2} = -1$$

Show these two solutions match $\begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$ and $\begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$

$\sinh t$ at $t=0$ is $\frac{e^0 - e^{-0}}{2} = 0$ $\cosh t$ at $t=0$ is $\frac{e^0 + e^{-0}}{2} = 1$

$\frac{d}{dt}\sinh t = \cosh t$. so $y'(0) = 1$ for sinh.

$\frac{d}{dt}\cosh t = \sinh t$ " $y'(0) = 0$ for cosh.

Also useful : $\cosh^2 t = 1 + \sinh^2 t$.