MATH 23 WORKSHEET : Undetermined coefficients

10/17/07
Burnett

Consider $y'' + 3y' + 2y = g(t)$ (roots are what? : ; $g(t)$: driving)

i) for $g(t) = e^{-3t}$, guess the form of $Y(t)$ and solve for coefficient:

y'' + $3y'$ + $2y$

$Y'' =$ $Y' =$ $\xleftarrow{d/dt}$ $Y = ?$

ii) for $g(t) = e^{-t}$, guess the form of $Y(t)$ and try as above:

What's gone wrong? Why?

Guess a better $Y(t)$ form [hint: think back to repeated roots ...]

$Y(t) = A \ldots$

Try it:

y'' + $3y'$ + $2y$ = e^{-t}

$Y'' =$ $Y' =$ $\xleftarrow{d/dt}$ $Y =$

Solve for A:

iii) Bonus: solve $y'' + 2y' + y = e^{-t}$.

SOLUTIONS

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Consider $y'' + 3y' + 2y = g(t)$ (driving)

roots are what? : $-1, -2$ — watch out!

i) for $g(t) = e^{-3t}$, guess the form of $Y(t)$ and solve for coefficient

$$y'' + 3y' + 2y = e^{-3t}$$

$Y = Ae^{-3t}$ $\xrightarrow{d/dt}$ $Y' = -3Ae^{-3t}$ $\xrightarrow{d/dt}$ $Y'' = +9Ae^{-3t}$

coeffs in e^{-3t}: $9A - 9A + 2A = 1$. $A = 1/2$.

so $Y(t) = \frac{1}{2}e^{-3t}$ is a particular solution

ii) for $g(t) = e^{-t}$, guess the form of $Y(t)$ and try as above:

$Y = Ae^{-t}$ — $Y' = -Ae^{-t}$ — $Y'' = Ae^{-t}$.

$$A - 3A + 2A = 1. \quad \text{ie } 0A = 1$$

$0 = 1$ no such A!

What's gone wrong? Why?

e^{-t} was a solution to the homogeneous eqn. $y'' + 3y' + 2y = 0$, so LHS vanished as no use in matching the driving.

Note: e^{-2t} would also cause problem.

Guess a better $Y(t)$ form [hint: think back to repeated roots ...]

$$Y(t) = Ate^{-t}$$

Try it:

$$y'' + 3y' + 2y = e^{-t}$$

$Y = Ate^{-t}$ $\xrightarrow{d/dt}$ $Y' = Ae^{-t} - Ate^{-t}$ $\xrightarrow{d/dt}$ $Y'' = -Ae^{-t} - Ae^{-t} + Ate^{-t}$

Solve for A: coeffs of te^{-t} are: $A - 3A + 2A = 0$ tells you nothing, but is consistent

coeffs of e^{-t}: $-2A + 3A = 1$ so $A = 1$. ✓

ii) Bonus: solve $y'' + 2y' + y = e^{-t}$: since e^{-t}, te^{-t} already known, solns, use $Y = t^2e^{-t}$