

MATH 23 WORKSHEET : Row reduction.

Bryant
10/24/05

To invert $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ -3 & 1 & 2 \end{bmatrix}$ write I next to it & perform row operations, to get $A \rightarrow I$

- i) multiply a row by a number
- ii) add multiple of row to another row
- iii) Swap 2 rows.

Start.

$$\begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ \\ \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{how much of } R_1 \text{ to} \\ \text{add to cancel the } -2? \\ \text{same for the } -3 \text{ in } R_3. \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array} \right]$$

fill in rest., including RHS!

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

now make 2nd entry in R_3 go away by adding/subtracting R_2 from it.

make sure all diag. entries on LHS are 1.

Now add some of R_3 to R_2 to remove 3rd entry on R_2 :

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

Now make the -1 go away here.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

↑ Here you'll find A^{-1} .

can check by multiplying $A^{-1}A$ or AA^{-1} !

MATH 23 WORKSHEET : Row reduction for inverse

Bennett
10/27/05

SOLUTIONS

To invert $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ -3 & 1 & 2 \end{bmatrix}$ write I next to it & perform row operations, to get $A \rightarrow I$

- i) multiply a row by a number
- ii) add multiple of row to another row
- iii) Swap 2 rows.

Start.

$$\begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

how much of R_1 to add to cancel the -2? same for the -3 in R_3 .

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 + 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2 & 1 & 0 \\ 0 & -2 & 2 & 3 & 0 & 1 \end{array} \right]$$

fill in rest, including RHS!

$$R_3 - R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array} \right]$$

now make 2nd entry in R_3 go away by adding/subtracting R_2 from it.

make sure all diag. entries on LHS are 1.

Now add some of R_3 to R_2 to remove 3rd entry on R_2 :

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array} \right]$$

rescale (multiply) R_2 by $-1/2$ after you subtract R_3 easily (tricky, you could do it in two stages.)

Now make the -1 go away here:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 1/2 \\ 0 & 1 & 0 & -1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array} \right] \quad R_1 + R_2$$

$I \quad A^{-1}$
Here you'll find A^{-1} .

can check by multiplying $A^{-1}A$ or AA^{-1} !