WORKSHEET:

Barnett
10/3/05 &
10/8/07

2^{nd}-order linear ODEs — const. coeffs. homogeneous case

a) Consider $y'' - y = 0$

Find a func. whose 2^{nd} deriv. is itself: $y_1(t) =$

Find another (different!) one [Hint: what if $y \xrightarrow{d/dt} -y \xrightarrow{d/dt} y$?]

$y_2(t) =$

b) Using your y_1 & y_2, evaluate $\frac{d^2}{dt^2}[5y_1(t) - 3y_2(t)]$

So does $5y_1(t) - 3y_2(t)$ satisfy $y''-y=0$?

Does $c_1 y_1(t) + c_2 y_2(t)$ satisfy it for all c_1, c_2 ?

c) Choose c_1, c_2 to match this initial condition:

$$\begin{cases} y(0) = 3 \\ y'(0) = 4 \end{cases}$$

Could you match any IC y_0, y_0' ?

Why? [Hint: find general formula for c_1, c_2]

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2^{nd}-order linear ODEs — const. coeffs. homogeneous case

a) Consider $y'' - y = 0$

Find a func. whose 2^{nd} deriv. is itself: $y_1(t) = e^t$

Find another (different!) one [Hint: what if $y \xrightarrow{d/dt} -y \xrightarrow{d/dt} y$?]

note: $\sin t \xrightarrow{d/dt} \cos t \xrightarrow{d/dt} -\sin t$ ← wrong sign!

$y_2(t) = e^{-t}$.

b) Using your y_1 & y_2, evaluate $\frac{d^2}{dt^2}[5y_1(t) - 3y_2(t)]$

So does $5y_1(t) - 3y_2(t)$ satisfy $y'' - y = 0$?

$\frac{d^2}{dt^2}(5e^t - 3e^{-t}) = \frac{d}{dt}(5e^t + 3e^{-t}) = (5e^t - 3e^{-t})$, ie itself again.

$\Rightarrow$ yes, ODE is satisfied!

Does $c_1y_1(t) + c_2y_2(t)$ satisfy it for all c_1, c_2 ?

Yes since 5, −3 were arbitrary #s.

c) Choose c_1, c_2 to match this initial condition:

$$\begin{cases} y(0) = 3 \\ y'(0) = 4 \end{cases}$$

$y(0) = c_1e^0 + c_2e^{-0} = \boxed{c_1 + c_2 = 3}$

$y' = c_1e^t - c_2e^{-t}$ so $y'(0) = c_1e^0 - c_2e^{-0} = \boxed{c_1 - c_2 = 4}$

due to $\frac{d}{dt}(e^{-t})$.

solve 2 simultaneous eqns.

$c_1 = 7/2$

$c_2 = -1/2$

) Could you match any IC y_0, y_0' ?

Yes

Why? [Hint: find general formula for c_1, c_2]

$\left.\begin{array}{l} c_1 + c_2 = y_0 \\ c_1 - c_2 = y_0' \end{array}\right\} \rightarrow \begin{array}{l} c_1 = \frac{y_0 + y_0'}{2} \\ c_2 = \frac{y_0 - y_0'}{2} \end{array}$