

# Math 22: Linear Algebra. PRACTISE MIDTERM 2

No calculators. Please answer on this sheet. Your NAME:

1. [REALLY IMPORTANT ONE!]

(a) find the determinant of  $\begin{bmatrix} 2 & 7 & 1 \\ 2 & 7 & 2 \\ 4 & 10 & 0 \end{bmatrix}$ , stating which method you used.

(b) find the eigenvalues *and eigenvectors* of  $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$

2. [REALLY IMPORTANT TOO!] Find a basis for Col  $A$  and Row  $A$  of

the matrix  $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$

Prove that if  $H$  is any subspace of a vector space  $V$ , then  $\dim H \leq \dim V$ . [Hint: what is the largest number of lin. indep. vectors possible in  $V$ ?]

3. Compute (without using row swaps) the  $LU$  decomposition of

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 0 & -1 \\ 4 & 2 & 2 \end{bmatrix}$$

[Hint: you may want to check that  $A = LU$  once done]

$$L = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \quad U = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

Give a reason why the  $LU$  decomposition is useful in the real world.

4. Consider the set  $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{array}{l} x + y - z = 0 \\ 2y + z = 0 \\ -x - 3y = 0 \end{array} \right\}$

(a) Can  $W$  be written as Col  $A$  or Row  $A$  or Nul  $A$  for some matrix  $A$ ? Use this, or another method, to *prove* that  $W$  is a subspace of  $\mathbb{R}^3$ .

(b) Find a basis for  $W$ .

What is  $\dim W$  ?

5. Say  $H$  is some subspace of  $\mathbb{R}^3$ , for which a basis has been found to be

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

(a) Find  $\mathbf{x}$  given that  $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

(b) Find  $[\mathbf{x}]_{\mathcal{B}}$ , the coordinates of the point  $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$  in the  $\mathcal{B}$  basis.  
(PS you should find that  $\mathbf{x}$  is in  $H$ ).

$$[\mathbf{x}]_{\mathcal{B}} =$$

6. (a) True/false?  $\mathbb{R}^2$  is a subspace of  $\mathbb{R}^3$ ? Why?
- (b) Define isomorphism.
- (c) True/false? If  $\mathbf{x}$  is an eigenvector of a  $n \times n$  matrix  $A$  with eigenvalue  $\lambda_1$ , then it is possible that  $\mathbf{x}$  is also an eigenvector of  $A$  with eigenvalue  $\lambda_2 \neq \lambda_1$ ? Explain.
- (d) Does the set of functions  $\{2 + 4t^2, t + 3t^2, t + 2t^2\}$  form a basis for the polynomials  $\mathbb{P}_2$ ? If so, explain the criteria which you tested. If not, what dimension subspace do they span?

Practise questions include the following. You could make your own practise exam by picking 5–6 of them, one from each major section, and throwing in some true/false questions from the supplementary exercises. I apologize if a couple duplicate HW questions. Also, if one seems too hard for an exam, or you are stuck how to do it, email me to ask.

**2.5:** 2, 9.

Chapter 2 supplementary exercises (p. 183): 1, 3.

**3.1:** 7, 8, 10, 12.

**3.2:** 5, 6, 7, 15, 16, 21, 22, 24, 25.

Chapter 3 supplementary exercises (p. 211): 1, 5 (look for best row or col each time), 14 ab.

“PP” means Practise Problem, which have worked solutions after the Exercises.

**4.1:** PP1, 9, 14, 17.

**4.2:** 15, 23.

**4.3:** 11, 13.

**4.4:** 1, 5, 21.

**4.5:** 11, 13.

**4.6:** PP1-4, 1 (good summary).

Chapter 4 supplementary exercises: 1 (good but some are hard), 7 (use Rank Theorem).

**5.1:** 1, 9, 17, 23, 25.

**5.2:** 3, 11, 20.