MATH 22 WORKSHEET : Span

6/27/06

Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$ ← h is some number.

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ ← k is some number. in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?

Row reduce the augmented matrix to Echelon Form :

- treat h & k as you would usual numbers
- scale a row to remove fractions.

What set of h, k makes linear system

a) consistent & unique?

b) consistent but not unique?

MATH 22 WORKSHEET : Span

6/27/06

Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\quad \vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$ $\quad \vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$ ← h is some number.

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?

← k is some number.

Row reduce the augmented matrix to Echelon Form:

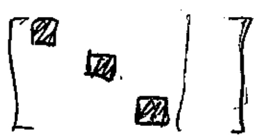
- treat h & k as you would usual numbers
- scale a row to remove fractions.
  - actually, not important.

$$\begin{bmatrix} 0 & 1 & -4 & 1 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & h & k \end{bmatrix} \sim \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 5 & -8 & h & k \end{bmatrix}$$

$R_3 \to R_3 - 5R_1$ $\qquad$ $R_3 \to R_3 + \frac{1}{2}R_2$

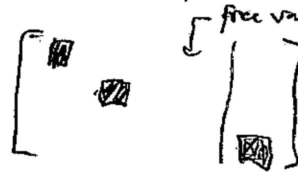
$$\sim \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 0 & -1/2 & h-5 & k-5/2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & h-7 & k-2 \end{bmatrix}$$ Echelon Form.

Examine pivot positions (depending on h, k):



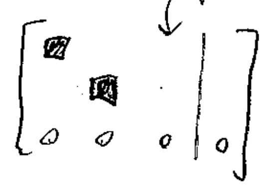
$h \neq 7$, any k.
consistent, unique

free var.



$h = 7$, $k \neq 2$
inconsistent.

free var.



$h = 7$, $k = 2$
consistent, non-unique.

What set of h, k makes linear system

a) consistent & unique? $h \neq 7$, k = anything

b) consistent but not unique? $h = 7$, $k = 2$.

It's all about the pivots.