MATH 22 WORKSHEET: standard matrix. 7/5/06

For each described operation, find standard matrix A, and whether T is onto and one-to-one

what size?

a) $T(x_1, x_2) = (3x_1, -2x_1 + x_2, -x_2)$

$T: \mathbb{R}^n \to \mathbb{R}^m$ (what are n, m?).

$A =$

onto?

one-to-one?

b) T is reflection about line $x_2 = x_1$

$(T: \mathbb{R}^2 \to \mathbb{R}^2)$.

$A =$

onto?

one-to-one?

c) $T: \mathbb{R}^3 \to \mathbb{R}^2$

projects the point (x, y, z) down vertically onto the (x, y) plane

(the shadow of a point under the midday sun).

$A =$

onto?

one-to-one?

MATH 22 WORKSHEET: standard matrix. — answers. 7/5/06

For each described operation, find standard matrix A, and whether T is <u>onto</u> and <u>one-to-one</u>

columns given by

$$\left[T(\vec{e}_1)\ T(\vec{e}_2)\right]$$

what size?

a) $T(x_1, x_2) = (3x_1, -2x_1 + x_2, -x_2)$

$T: \mathbb{R}^n \to \mathbb{R}^m$ (what are n, m?).

$$A = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & -1 \end{bmatrix} \sim \begin{bmatrix} \blacksquare & * \\ & \blacksquare \\ & \end{bmatrix}$$

2 pivots

2 vectors cannot span $\mathbb{R}^3$ since would need a pivot in each of 3 rows for this.

onto? No, since $A\vec{x} = \vec{b}$ not consistent for all $\vec{b}$ in $\mathbb{R}^3$.

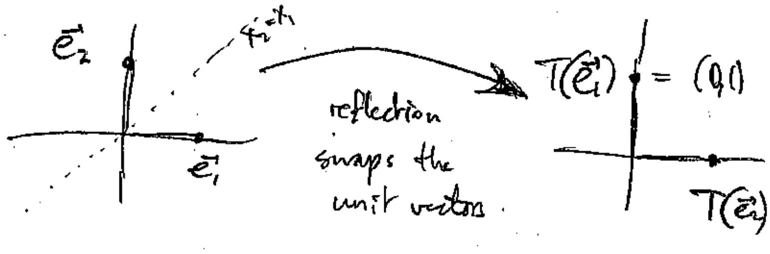
one-to-one? Yes, since when $A\vec{x} = \vec{b}$ is consistent, it is <u>unique</u> ($\vec{b}$ is image of <u>single</u> $\vec{x}$)

since no free vars, pivot in every col.

b) T is reflection about line $x_2 = x_1$

$(T: \mathbb{R}^2 \to \mathbb{R}^2)$.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



onto? Yes since pivot in every row.

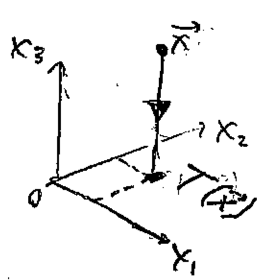
one-to-one? Yes since there are no free vars in $A\vec{x} = \vec{b}$.

could also answer geometrically.

c) $T: \mathbb{R}^3 \to \mathbb{R}^2$

projects the point (x, y, z) down vertically onto the (x, y) plane

(the shadow of a point under the midday sun).



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \text{ in REF}$$

onto? Yes since pivot in every row

one-to-one? No, since in $A\vec{x} = \vec{b}$, x_3 is free var, not unique.