HW 7 Math 22

4.6 14 (2 points) If A is 4×3, its rows are in $\mathbb{R}^3$ and there can be at most 3 linearly independent vectors in such a set. Also, it cannot have more than 3 linearly independent rows. because there are only 3 rows.

18 (3 points) a. False. See "warning" after proof of Theorem 6 in section 4.3

b. False. See "warning" after example 2.

c. True. See the remark in the proof of the Rank Theorem.

d. True. See the paragraph before Example 4.

e. True. See Theorem 13.

24 (2 points) 1° Yes. In this case, there are no free-variables, so by the Rank Theorem, the rank of A must equal the number of columns.

2° No. The rank of A cannot exceed 6, so Col A must be a proper subspace of $\mathbb{R}^7$.

$\therefore$ There exist vectors in $\mathbb{R}^7$ that are not in Col A

For such right-hand sides, $Ax=b$ will have no solution.

5.1 2 (2 points) Yes. $(A-\lambda I)x=0$ has a non-trivial solution.

14 (2 points) $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

24 (2 points) General form: $\begin{bmatrix} a & b \\ c & a \end{bmatrix}$, (+1 bonus) where $b\cdot c=0$

5.2 7 (2 points) Characteristic equation: $\lambda^2 - 9\lambda + 32$. No real eigenvalues.

10 (2 points). $-\lambda^3+14\lambda+12$

5.2 16 (2points) 5, 1, 1, −4.

19 (2points) Note that the given equation holds for all λ. Let $\lambda = 0$ $\therefore \det(A) = \lambda_1 \lambda_2 \cdots \lambda_n$.

A. 1. n^3 ← you should get rough ratios of 27 between 100 & 300, about same from 300 & 1000. If they vary by up to factor 2, this is fine.

2. $A + A^T$ is a random symmetric matrix

3. About $(10^3)^3$ times longer than the $n = 10^3$ case, ie., $\approx 10^{10}$ sec ≈ 30 years.

$O(n^3)$ is same scaling as $O(n^3) = \frac{2n^3}{3}$ for row reduction

Some of you found $n = 300$ was only 10 times slower than $n = 100$, concluding $O(n^2)$. Sorry about this!

But the ratio $n = 10^3$ to $n = 300$ should be close to 30 for all computers.