

- 1.9 ²⁴ (3 points) a. False, see the paragraph preceding Example 2.
 b. True, see Theorem 10.
 c. True, see Table 1.
 d. False, see def. of one-to-one
 e. True, see the solution of Example 5.

2.1 ¹² (2 points) A suitable column for θ is any multiple of $(2, 1)$, e.g., $B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

- 2.2 ¹⁰ (3 points) a. False, the product of inverses should be in the reverse order, e.g., Theorem 6(b).
 b. True, by Theorem 6(a)
 c. True, by Theorem 4.
 d. True, by Theorem 7.
 e. False. This is a misstatement of Theorem 7

2.2 ²³ (2 points). $AX=0$ has only the trivial solution \Rightarrow there are no free variables in $AX=0$
 Thus each column of A is a pivot column.

³³ (3 points) $A^{-1} = B = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix}$ ← note: $\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$
 is an integration matrix (discrete, i.e. summing) — try it.
 ← This is differentiation matrix.

2.3 ⁶ (2 points) Not invertible. The matrix you reduce to $\begin{bmatrix} 1 & -4 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$, and is not row equivalent to I_3 .

so you've proved the inverse of integration is differentiation, (discretely).

2.3 ¹² (2 points) a. True. Statement (k) of the I.M.T is true \Rightarrow (j) is true

b. True. $\dots (e) \dots \Rightarrow (b)$ is true.

c. True. See remark following the proof of the I.M.T.

d. False. If A is any $n \times n$ matrix, the linear transformation $x \mapsto Ax$ maps \mathbb{R}^n to \mathbb{R}^n , yet not every such matrix has n pivot positions.

e. True, by I.M.T

2.5 ² (2 points) $2y = b \Rightarrow y = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, $Ux = y \Rightarrow x = \begin{bmatrix} 1/4 \\ 7 \end{bmatrix}$.

10 (2 points) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} \begin{bmatrix} -5 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$

14 (2 points) Row reduce $[A \ I]$

Bonus: (1 point) The inverse of an upper triangle matrix is an upper triangle matrix.

3.1 ² (2 points) 2

38 (2 points) $\det kA = k^2 \det A$