Some solutions

Problem 1 on the web.

Let $\{1,2,\ldots,k\}$ be the set of k students, and let U_j be the event "student j sits in the same chair after tea as before." Then

$$P(U_j) = \frac{\text{\# of seating arrangements with j in the same seat after tea as before}}{\text{total \# of seating arrangements}}$$

$$= \frac{(n-1)(n-2)(n-3)\ldots(n-k+1)}{n(n-1)(n-2)\ldots(n-k+1)} = \frac{(n-1)_{k-1}}{(n)_k} = \frac{1}{n},$$

where $(n)_r = n(n-1)(n-2)\ldots(n-r+1)$. (There are r factors in the product.) Similarly,

$$P(U_i \cap U_j) = \frac{(n-2)_{k-2}}{(n)_k} = \frac{1}{n(n-1)} = \frac{1}{(n)_2}$$

since the number of seating arrangements with students i and j in the same seats after tea as before is $(n-2)(n-3)\ldots(n-k+1) = (n-2)_{k-2}$. And $P(U_i \cap U_j \cap U_l) = \frac{1}{(n)_3}$, etc. So by the inclusion-exclusion principle, the probability that some student is in the same chair after tea as before is

$$P(U_1 \cup \ldots \cup U_k) = \binom{k}{1}\frac{1}{n} - \binom{k}{2}\frac{1}{(n)_2} + \binom{k}{3}\frac{1}{(n)_3} - \ldots + (-1)^{n-1}\binom{k}{k}\frac{1}{(n)_k}.$$

Therefore the probability that no student is sitting in the same seat after tea as before is

$$1 - P(U_1 \cup \ldots \cup U_k) = 1 - \binom{k}{1}\frac{1}{n} + \binom{k}{2}\frac{1}{(n)_2} - \binom{k}{3}\frac{1}{(n)_3} + \ldots + (-1)^{n}\binom{k}{k}\frac{1}{(n)_k}.$$

3.2, 34. With E_k as suggested in the hint,

$$P(E_k) = b(m, \frac{1}{n}, 0) = \left(\frac{n-1}{n}\right)^m,$$

where success is defined as getting the kth player's picture in a box of Wheaties. The probability of getting neither the jth nor the kth player's picture in a box is $\frac{n-2}{n}$, so with success now defined as this last event, we have

$$P(E_k \cap E_j) = b(m, \frac{n-2}{n}, m) = \left(\frac{n-2}{n}\right)^m.$$

The probabilities of the other intersections of the E_k are found similarly. Now use the inclusion-exclusion principle.

3.2, 18. Assume that all the students do the test by flipping coins. Then, given a student,

$$P(he\ gets \leq 2\ questions\ correct) = \sum_{i=0}^{2} b(10, \frac{1}{2}, i) = \frac{7}{128},$$

and

$$P(he\ gets\ no\ questions\ correct) = b(10, \frac{1}{2}, 0) = \frac{1}{1024}.$$

So, of the 340 students,

$$P(at\ least\ one\ has\ \ \leq 2\ correct) =$$

$$1 - P(none\ with \leq 2\ correct) = 1 - b(340, \frac{7}{128}, 0) \approx 1 - 5 \cdot 10^{-9},$$

which is very close to 1. So Prosser is correct. Also,

$$P(at\ least\ one\ with\ none\ correct) =$$

$$1 - P(none\ with\ none\ correct) = 1 - b(340, \frac{1}{1024}, 0) \approx 1 - .7 < \frac{1}{2}.$$

So Crowell is wrong.