

1. Copy the following limit computations onto a separate sheet of paper. Justify each labeled step (in complete sentences) using only the limit laws and theorems found in sections 2.3 and 2.6. In part (b.) complete the computation.

(a.) (Justify each labeled step.)

$$\begin{aligned}
 \lim_{x \rightarrow -\infty} \frac{4x^2 - 3x + 1}{7x^2 + x - 1} &\stackrel{\text{A}}{=} \lim_{x \rightarrow -\infty} \frac{\left(\frac{4x^2 - 3x + 1}{x^2}\right)}{\left(\frac{7x^2 + x - 1}{x^2}\right)} \\
 &= \lim_{x \rightarrow -\infty} \frac{4 - 3 \cdot \frac{1}{x} + \frac{1}{x^2}}{7 + \frac{1}{x} - \frac{1}{x^2}} \\
 &\stackrel{\text{B}}{=} \frac{\lim_{x \rightarrow -\infty} \left(4 - 3 \cdot \frac{1}{x} + \frac{1}{x^2}\right)}{\lim_{x \rightarrow -\infty} \left(7 + \frac{1}{x} - \frac{1}{x^2}\right)} \\
 &\stackrel{\text{C}}{=} \frac{\lim_{x \rightarrow -\infty} 4 - 3 \cdot \lim_{x \rightarrow -\infty} \frac{1}{x} + \lim_{x \rightarrow -\infty} \frac{1}{x^2}}{\lim_{x \rightarrow -\infty} 7 + \lim_{x \rightarrow -\infty} \frac{1}{x} - \lim_{x \rightarrow -\infty} \frac{1}{x^2}} \\
 &\stackrel{\text{D}}{=} \frac{4 - 3 \cdot 0 + 0}{7 + 0 - 0} \\
 &= \frac{4}{7}
 \end{aligned}$$

(b.) (Justify each labeled step and complete the computation.)

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \frac{-2x^2 + 3x - 7}{x + 1} &\stackrel{\text{A}}{=} \lim_{x \rightarrow \infty} \frac{\left(\frac{-2x^2 + 3x - 7}{x}\right)}{\left(\frac{x + 1}{x}\right)} \\
 &= \lim_{x \rightarrow \infty} \frac{-2x + 3 - 7 \cdot \frac{1}{x}}{1 + \frac{1}{x}} \\
 &\stackrel{\text{B}}{=} \frac{-2 \cdot \lim_{x \rightarrow \infty} x + \lim_{x \rightarrow \infty} 3 - 7 \cdot \lim_{x \rightarrow \infty} \frac{1}{x}}{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x}} \\
 &\stackrel{\text{C}}{=} \frac{-2 \cdot \lim_{x \rightarrow \infty} x + 3 - 7 \cdot 0}{1 + 0} \\
 &= -2 \cdot \lim_{x \rightarrow \infty} x + 3 \\
 &= \underline{\hspace{1cm}}
 \end{aligned}$$

2. Using the definition of the derivative¹, compute the derivatives of the following functions at the value $x = 1$ (in other words find $f'(1)$ and $g'(1)$).

(a.) $f(x) = x^2 + 3x - 1$

(b.) $g(x) = \frac{x+1}{2x-1}$

3. Using the definition of the derivative, repeat problem 2. for an arbitrary value of x (in other words, find $f'(x)$ and $g'(x)$).

¹Whenever we say “using the definition of the derivative” we mean that you must actually compute one of the limits: $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ or $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$.