

Math 12, Fall 2007

Lecture 24

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Outline

- 1 Review and overview
 - Last class
- 2 Today's material
 - Surface integrals
- 3 Next class

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- Parameterized surfaces
- Surface Area

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 - **Surface integrals**
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Integrals over surfaces

Let S be a parameterized surface with parameter domain D and $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a function whose domain contains an open set which includes S .

$$\iint_S f(x, y, z) \, dS = \iint_D f(x, y, z) |\vec{N}| \, dA$$

Integrals over surfaces

A comparison of integrals

1

$$\int_a^b ds = b - a$$

vs.

$$\int_a^b |\vec{r}'(t)| dt = \int_C ds = \text{Length}(C)$$

2

$$\iint_D dA = \text{Area}(D), D \subset \mathbb{R}^2$$

vs.

$$\iint_S |\vec{N}| dA$$

S a parameterized surface

Integrals over surfaces

A comparison of integrals

1

$$\iiint_R dv = \text{Volume}(V)$$

VS

$$\iint_D f(x, y) dA$$

VS

$$\iint_S f(x, y, z) dS$$

Examples



$$\iint_S yz \, dS$$

S is given by

$$x = u^2, y = u \sin(v), z = u \cos(v), 0 \leq u \leq 1, 0 \leq v \leq \frac{\pi}{2}.$$



$$\iint_S \sqrt{1 + x^2 + y^2} \, dS$$

where S is the helicoid given by

$$\vec{r}(u, v) = \langle u \cos(v), u \sin(v), v \rangle, 0 \leq u \leq 1, 0 \leq v \leq \pi$$

Integrals of vectors fields over surfaces

Recall for line integrals we had integrals of functions

$$\int_C f(x, y, z) ds$$

and integrals of vector fields

$$\int_C \vec{F} \cdot d\vec{r}$$

Orientation

An *orientation* for a surface S is a choice of continuous unit normal vector.

- Idea: let $\vec{N} = \vec{r}_u \times \vec{r}_v$ and check if $\vec{N}/|\vec{N}|$ is continuous.
- Example of non-orientable surface: Möbius band
- A closed surface is *positively oriented* if it is equipped with its outward pointing normal.
- Example: Find positive and negative orientations on a sphere.

Integrals of vectors fields over surfaces

If \vec{F} is a continuous vector field defined on an oriented surface S with unit normal vector \vec{n} , then

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} \, dS$$

This integral is also called a flux integral.

Examples

- $\vec{F} = \langle x, -z, y \rangle$, S is the part of the sphere of radius 2 in the first octant.
- $\vec{F} = \langle x, 2y, 3z \rangle$ where S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$

Work for next class

- Reading: 17.8
- Webwork: f07hw23