

Math 12, Fall 2007

Lecture 17

Scott Pauls ¹

¹Department of Mathematics
Dartmouth College

11/5/07

Outline

- 1 Review and overview
 - Last class
- 2 Today's material
 - Triple Integrals: coordinate transformations
- 3 Next class

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Triple integrals

- Triple integrals allow us to integrate over solid regions in \mathbb{R}^3
- Fubini's theorem and the theory of iterated integrals still apply.
- The main new difficulty is visualizing and parametrizing three dimensional solids.

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Integrals of functions of three variables

- Polar coordinates: Change any two variables via polar coordinates:

$$u = r \cos(\theta), \quad v = r \sin(\theta), \quad dudv = r dr d\theta$$

This use of polar coordinates is also called cylindrical coordinates

- Spherical coordinates:

$$x = \rho \sin(\phi) \cos(\theta), \quad y = \rho \sin(\phi) \sin(\theta), \quad z = \rho \cos(\phi)$$

$$dV = \rho^2 \sin(\phi) d\rho d\theta d\phi$$

Corrdinate transformations

- 1 Coordinate transformations help describe complicated regions in a simpler way
- 2 Circles, Cylinders \rightarrow polar/cylidrical coordinates
- 3 Spheres \rightarrow spherical coordinates

Examples

1 Find the volume enclosed by the sphere of radius r .

2 Find

$$\iiint_R z \, dV$$

where R is the region that lies between $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ in the first octant.

3 Find the volume of the solid that lies above the xy -plane, inside the sphere of radius two centered at the origin and below the cone $z = \sqrt{x^2 + y^2}$.

4 A torus is a surface generated by taking a circle of radius r centered at $(x_0, 0)$ (with $x_0 > r$) and rotating it (in three space), about a line. How can we describe a torus most efficiently? What is its volume?

Work for next class

- Read 16.8,17.1-17.2
- f07hw19