Math 11 Practice Exam Problems

Disclaimer: This set of problems is meant neither to indicate the length nor composition of the actual exam. These are merely problems which were considered for inclusion on your exam, but for one reason or another were rejected. On the other hand, they should provide some flavor of the type of problems we considered.

1. Find the volume of the solid generated by revolving, about the line $x = \pi$, the region in the first quadrant bounded by $y = \sin(x)$, $y = 0$ and $x = \pi/2$. Show that this is the same volume as taking the region bounded by the sine and the x-axis between $x = \pi/2$ and $x = \pi$, and revolving it around the y-axis.

2. Determine whether the following integrals converge or diverge.

   $$\int_0^\infty \frac{dx}{\sqrt[3]{x} + x^3} \qquad \int_1^\infty \frac{dx}{x + e^{2x}} \qquad \int_2^3 \frac{dx}{\sqrt{3-x}} \qquad \int_0^\infty \frac{x\,dx}{1+x^2}$$

3. Using Trapezoid rule, find the minimum number of subintervals n needed to approximate $\int_0^2 \arctan(x)\,dx$ with an error less than 10^{-8}. Recall that the error formula for the Trapezoid Rule says that if $|f''(x)|$ is bounded by a constant K on the interval $[a,b]$, then the error (E_n) involved in using n subintervals for the approximation of $\int_a^b f$ satisfies

   $$E_n \leq \frac{K(b-a)^3}{12n^2} = \frac{K(b-a)(\Delta x)^2}{12}, \quad \Delta x = (b-a)/n.$$

4. Compute the Taylor polynomial of degree 5 for $f(x) = \sqrt{x}$ at 4.

5. Determine whether the following sequences converge or diverge. If convergent, determine the value.

   $$a_n = \frac{n\cos(n)}{1+n^2} \qquad b_n = n\sin(1/n) \qquad c_n = (-1)^n e^{-n}$$

6. Determine whether the following series converge or diverge.

   $$\sum_{n=3}^\infty \frac{2^{2n+1}}{3^n} \qquad \sum_{n=1}^\infty \frac{n^2+3n-2}{5n^2+7n+2} \qquad \sum_{n=1}^\infty \arctan(n+1) - \arctan(n)$$

   $$\sum_{n=2}^\infty \frac{1}{n\ln(n)} \qquad \sum_{n=2}^\infty \frac{\ln(n)}{n^3} \qquad \sum_{n=1}^\infty n\sin(1/n)$$