

## Math 11 97F Practice Exam 1

**Disclaimer:** Below are the problems from last year's Math 11 exam, however it should be clearly understood that this set of problems is meant neither to indicate the form, length, nor the composition of this year's exam, but it does indicate the kinds of problems we consider reasonable.

1. (25) Evaluate the following integrals. *Show all work.*

a.  $\int [\ln(x)]^2 dx$

b.  $\int \frac{7x^2 - x - 1}{(x^2 + 1)(x - 2)} dx$

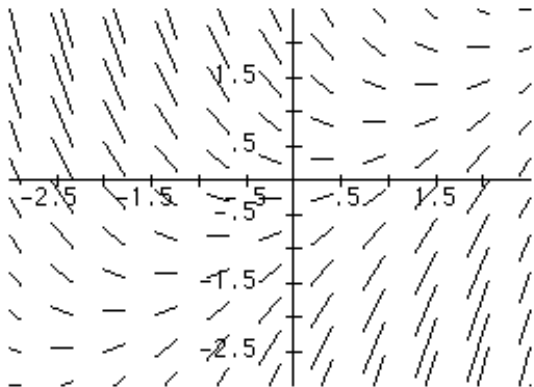
2. (25) Evaluate the following integrals. *Show all work.*

a.  $\int (\cos x + \sin x)^2 \cos(2x) dx$

b.  $\int \frac{x^2 dx}{\sqrt{x^2 - 1}}$

3. (15) Short Answer. No work need be shown, but no partial credit without some *correct* work.

- a. Consider the direction field below. The direction field corresponds to which differential equation? (circle the correct response).



(i)  $\frac{dy}{dx} = x + y$

(ii)  $\frac{dy}{dx} = x - y$

(iii)  $\frac{dy}{dx} = x^2 - y^2$

- b. What trigonometric substitution would be most appropriate to eliminate the radical in  $\int \frac{dx}{x^3 \sqrt{9x^2 - 4}}$ ? Do not evaluate the resulting integral.

- c. Suppose that in attempting to evaluate  $\int f(x) dx$ , you used the trigonometric substitution  $x = 4 \tan(\theta)$ . After integrating the resulting trigonometric integral, you obtained the answer  $3 \sin \theta + 7 \sec \theta - \theta + C$ . Convert your answer back to the original variable  $x$ .

4. (15) Short Answer. No work need be shown, but no partial credit without some *correct* work.

a. Determine the correct integrating factor to solve the differential equation  $(1 + x^2)\frac{dy}{dx} + 2xy = 3\sqrt{x}$ .

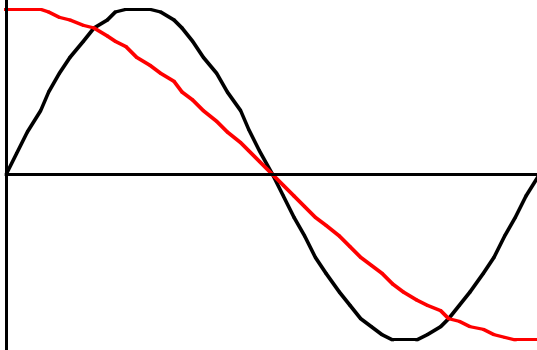
b. Find the general solution to the differential equation  $y'' - 4y' + 5y = 0$ .

c. Find the general solution to the differential equation  $y'' - 4y' + 4y = 0$ .

5. (10) Short Answer. No work need be shown, but no partial credit without some *correct* work.

a. A 5000 liter container is filled with 1000 liters of a brine solution into which 20 kg of salt has been dissolved. Two brine solutions are added to the container. The first solution contains .03 kg/l of salt and is added at 10 l/min, while the second contains .04 kg/l of salt and is added at 20 l/min. The solution is kept thoroughly mixed and drains from the container at a rate of 20 l/min. Derive (but do not solve) the differential equation which describes this system.

b. Determine (but do not solve) an integral (or integrals) which describes the area bounded by the curves  $y = \cos x$ ,  $y = \sin(2x)$ ,  $x = 0$  and  $x = \pi/2$ .



6. (10) Short Answer. No work need be shown, but no partial credit without some *correct* work.

a. Determine (but do not solve) an integral which describes the volume of the solid determined by revolving about the  $x$ -axis the region bounded by the curves  $y = 1 - (x - 1)^2$ ,  $y = 0$ ,  $x = 0$  and  $x = 2$ .

b. Simplify the expression  $(-2\sqrt{3} - 2i)^6$ . The answer should be expressed in simplest terms in the form  $a + bi$ .