Pattern replacement and factor replacement

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We report on recent work on an idea suggested by Jim Propp: consider two permutations in S_n to be equivalent if one can be obtained from the other by a sequence of pattern-replacing moves of prescribed form. For instance, we might allow any instance of 123 to be replaced by the same elements in the order 132, and vice versa. The natural questions are to count the number of equivalence classes, and to characterize or enumerate a particular equivalence class, most naturally the one containing the identity.

We consider the case where the exchanged elements are a pattern in the classical sense, but so far the more interesting enumerative results are for the case of factors, i.e. for substrings of consecutive elements. This setting recalls the well-known theory of Knuth equivalence.