Enumerating permutations containing few copies of 321 and 3412

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We will discuss the use of reduced decompositions in the enumeration of permutations containing few copies of 321 and 3412. The connections between reduced decompositions and the patterns 321 and 3412 were first studied by Billey, Jockusch and Stanley [1] and further studied by Tenner [4], [5].

The permutations which avoid both 321 and 3412 are known as boolean permutations and have been enumerated by West [6], Fan [3] and others.

Theorem 1. *The number of Boolean permutations in S_n is F_{2n-1} where F_k is the k^{th} Fibonacci number.*

At this conference two years ago, Daly [2], presented the enumeration for permutations containing 321 or 3412, but not both, precisely once.

Theorem 2. *The number of permutations in S_n which contain precisely one* 321 *and avoid* 3412 *is $\sum_{i=1}^{n-2} F_{2i}F_{2(n-i-1)}$ where F_k is the k^{th} Fibonacci number. This is also the number of permutations in S_{n+1} which contain precisely one* 3412 *and avoid* 321*.*

In this talk, we will show how to extend these results using reduced decompositions to the classes of permutations which avoid 3412 and contain precisely two copies of 321, precisely one copy of each pattern, or avoid 321 and contain precisely two copies of 3412. The situation becomes much more complicated for these cases, but exact formulas will be given for each case.

References

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