EXPANDING PERMUTATION STATISTICS AS SUMS OF PERMUTATION PATTERNS

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Any permutation statistic $f : \mathfrak{S} \to \mathbb{C}$ may be represented uniquely as a, possibly infinite, linear combination of (classical) permutation patterns: $f = \Sigma_\tau \lambda(\tau)\tau$. To provide explicit expansions for certain statistics, we introduce a new type of permutation patterns: Intuitively, such a pattern $p = (\pi, R)$ counts occurrences of the permutation pattern π with additional restrictions specified by R on the relative position of the entries of the occurrence. We show that, for any pattern $p = (\pi, R)$, we have $\lambda(\tau) = (-1)^{|\tau|-|\pi|} p^\star(\tau)$ where $p^\star = (\pi, R^c)$ is the pattern with the same underlying permutation as p but with complementary restrictions. We use this result to expand some well known permutation statistics, such as the number of left-to-right maxima, descents, excedances, fixed points, strong fixed points, and the major index.

This is joint work with Petter Brändén (KTH, Sweden).