A recurrence for $(1-23-4)$-avoiding permutations

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We show that the number $u(n)$ of permutations of $[n]$ that avoid the dashed pattern $1-23-4$ is given by $u(n) = \sum_{k=1}^{n} u(n,k)$, where the $u(n,k)$ satisfy the recurrence

$$u(n,k) = u(n-1,k-1) + k \sum_{j=k}^{n-1} u(n-1,j).$$

The proof relies on a bijection from the pattern-avoiding permutations to increasing ordered trees whose leaves, taken in preorder (aka walk-around order), are also increasing.