

## ANOTHER LOOK AT BIJECTIONS FOR PATTERN-AVOIDING PERMUTATIONS

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Bijections between  $S_n(321)$  and  $S_n(132)$  which preserve certain permutation statistics have received considerable attention in the literature. Robertson defined, through an iterative process, a remarkable natural bijection between these two sets. For any  $\sigma \in S_n(321)$  this bijection repeatedly removes the 132-pattern  $\sigma_x\sigma_y\sigma_z$  that is smallest in the lexicographic ordering of positions  $(x, y, z)$  and replaces it with the 321-pattern  $\sigma_y\sigma_z\sigma_x$  until  $\sigma \in S_n(132)$ . At the 2005 Integers conference he conjectured that this bijection,  $\Gamma$ , is fixed point preserving. Bloom and Saracino affirmed this conjecture and also showed that  $\Gamma$  preserves the number of excedances. It was also shown that  $\Gamma$  can also be defined such that at each iteration one removes the 132-pattern that is smallest either in terms of positions  $(x, y, z)$  or in terms of values  $(\sigma_x, \sigma_y, \sigma_z)$ . The proof depended on first showing that  $\Gamma(\sigma^{-1}) = (\Gamma(\sigma))^{-1}$  for all  $\sigma \in S_n(321)$ . Due to the iterative nature of  $\Gamma$  the proof of this fact was quite technical.

Here we will give a pictorial non-iterative definition of  $\Gamma$ . We accomplish this through a new combinatorial object, which we call a permutation template, that generalizes the notion of a permutation diagram. This new definition results in clearer and *independent* proofs that  $\Gamma$  commutes with inverses and preserves fixed points and excedances. Additionally, permutation templates provide the necessary structure to better understand why  $\Gamma$  may be defined in terms of any combination of position or value based 132-pattern replacements.