Math 8, Winter 2005

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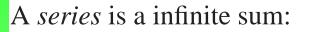
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$$\sum_{n=1}^{\infty} a_n$$

where $\{a_n\}$ is an infinite sequence.



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A *series* is a infinite sum:

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where $\{a_n\}$ is an infinite sequence.

• Define the m^{th} partial sum of an infinite series as:

$$s_m = \sum_{n=1}^m a_n$$





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A *series* is a infinite sum:

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where $\{a_n\}$ is an infinite sequence.

• Define the m^{th} partial sum of an infinite series as:

$$s_m = \sum_{n=1}^m a_n$$

• A series *coverges* if the sequence of partial sums $\{s_m\}$ converges. Otherwise the series *diverges*.



One easy consequence of this definition is the test for divergence:

Consider a series $\sum_{i=k}^{\infty} a_n$ and the sequence of summands, $\{a_n\}$. If a_n converges to $L \neq 0$ then $\sum_{i=k}^{\infty} a_n$ diverges.



The test for divergence

It is NOT TRUE that

If $a_n \to 0$ then $\sum_{i=k}^{\infty} a_n$ converges.

The harmonic series is a counterexample:

$$\frac{1}{n} \to 0$$

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but

$$\sum_{i=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$



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• We've seen two examples via improper integrals:

$$\sum_{i=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$

and

$$\sum_{i=1}^{\infty} \frac{1}{n^2} \quad \text{converges}$$



More examples

• Does

$$\sum_{i=2}^{\infty} \frac{1}{n^2 - n}$$

converge or diverge?

• Let a be a real number. For which values of r does

$$\sum_{i=0}^{\infty} ar^n$$

converge?



Geometric Series

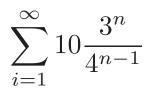
Suppose *a* is a real number and |r| < 1 then

$$\sum_{i=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$



Geometric series examples

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$$\sum_{i=0}^{\infty} -2\frac{1}{4^n}$$

$$\sum_{i=2}^{\infty} \frac{9^{n-1}}{10^{n-1}}$$



We've already used integrals to help determine the convergence of some series. We can now formalize this into a test:

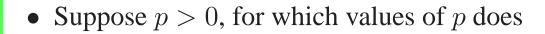
Suppose f is a continuous, positive, decreasing function defined for $1 \le x < \infty$ and let $a_n = f(n)$. Then

- If $\int_{1}^{\infty} f(x) dx$ converges, then $\sum_{i=1}^{\infty} a_{i}$ converges as well.
- If $\int_{1}^{\infty} f(x) dx$ diverges, then $\sum_{i=1}^{\infty} a_n$ diverges as well.











converge?

 $\sum_{i=1}^{\infty} \frac{\ln(n)}{n}$

