

```
[ > restart;
```

```
[ > with(student) :
```

Examples from class on 1/19/05 for numerically approximating integrals. For each of these, we will use $n=4$ and $n=8$ and each of the left, right, midpoint and trapezoidal rules.

```
[ > Int(y^3, y=0..1);
```

$$\int_0^1 y^3 dy$$

```
[ > f:=x->x^3;
```

$$f := x \rightarrow x^3$$

```
[ > Int(exp(y), y=0..1);
```

$$\int_0^1 e^y dy$$

```
[ > g:=x->exp(x);
```

$$g := x \rightarrow e^x$$

```
[ >
```

For $n=4$ we have the following x_i and $\Delta(x)$

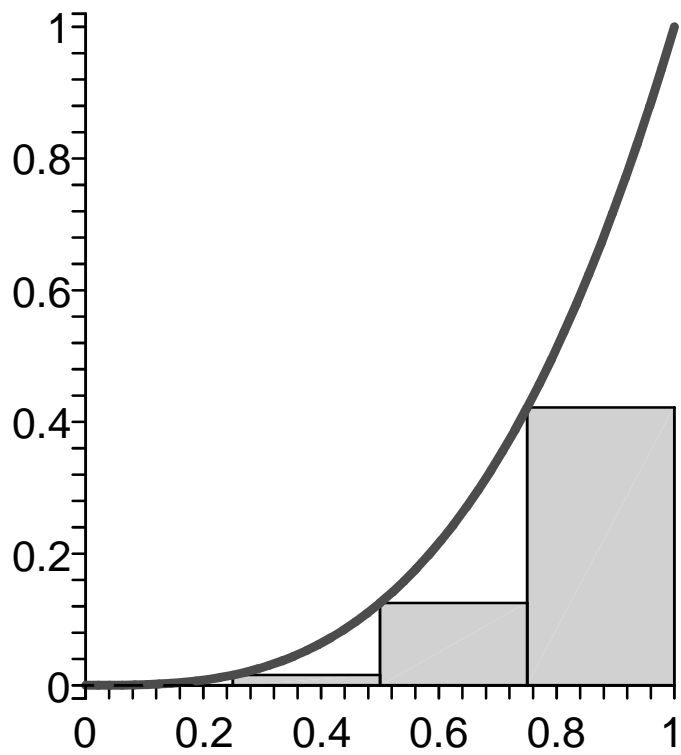
```
[ > x4 := [0, 1/4, 1/2, 3/4, 1];
```

$$x4 := \left[0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \right]$$

```
[ > Delta4 := 1/4;
```

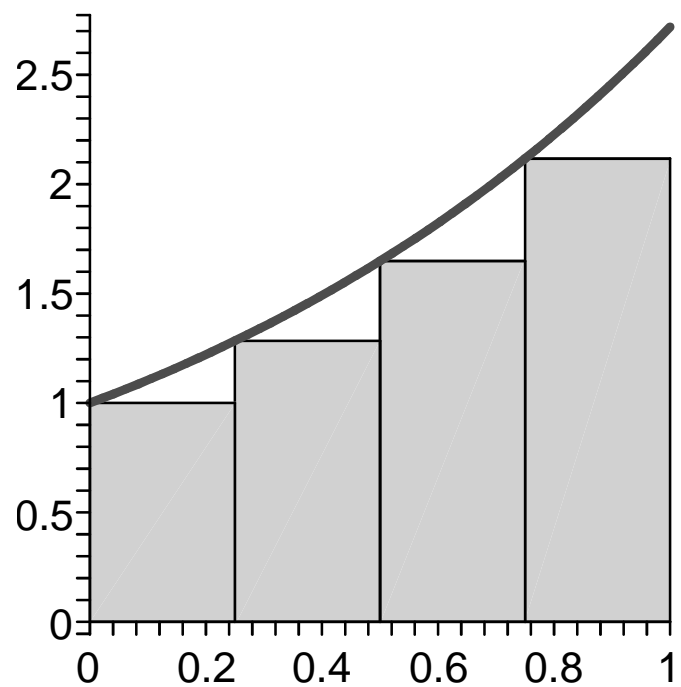
$$\Delta 4 := \frac{1}{4}$$

```
[ > leftbox(f(x), x=0..1, 4);
```



```
> L4f:=evalf(sum(f(x4[i])*Delta4,i=1..4));
      L4f:= 0.1406250000
```

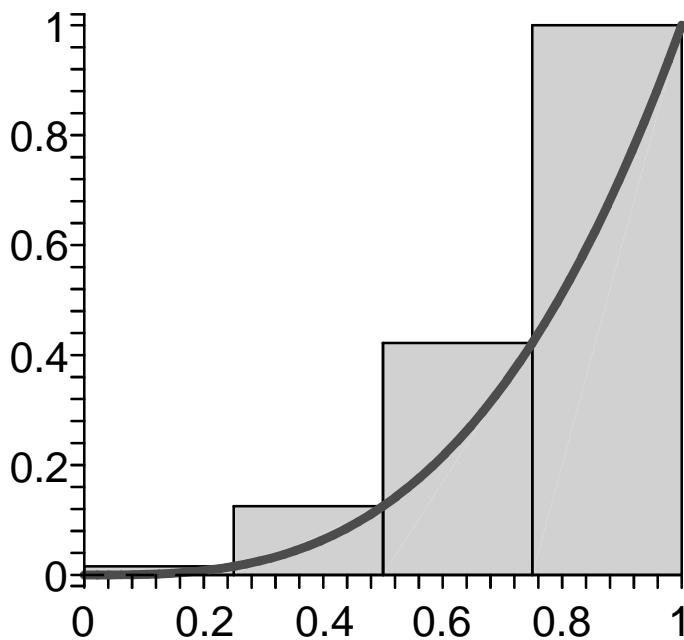
```
> leftbox(g(x),x=0..1,4);
```



```
> L4g:=evalf(sum(g(x4[i])*Delta4,i=1..4));  
L4g:=1.512436676
```

```
>
```

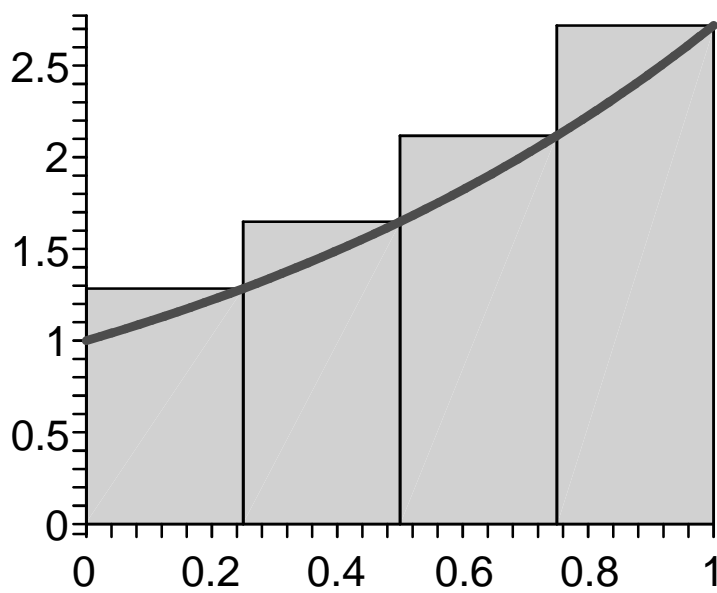
```
> rightbox(f(x),x=0..1,4);
```



```
> R4f:=evalf(sum(f(x4[i+1])*Delta4,i=1..4));  
R4f:=0.3906250000
```

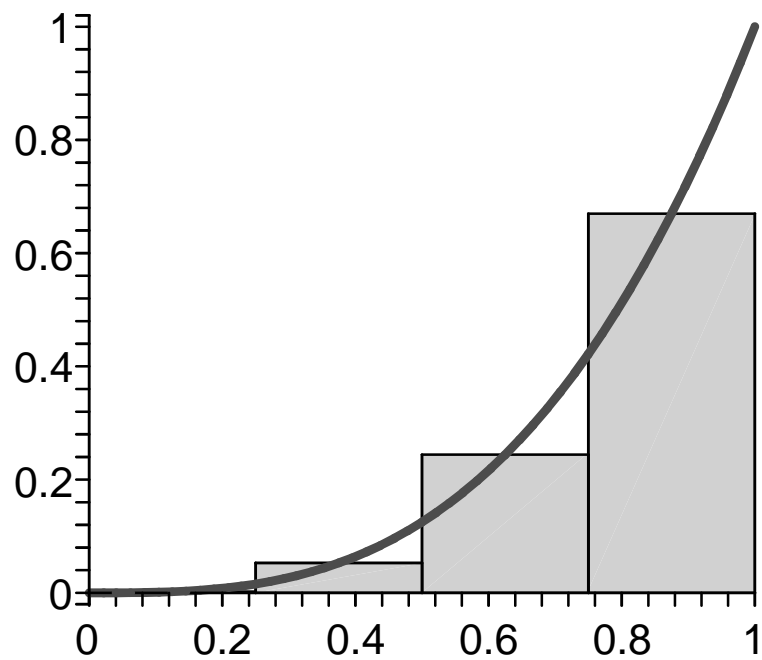
```
>
```

```
> rightbox(g(x),x=0..1,4);
```



```
> R4g:=evalf(sum(g(x4[i+1])*Delta4,i=1..4));  
R4g:=1.942007133
```

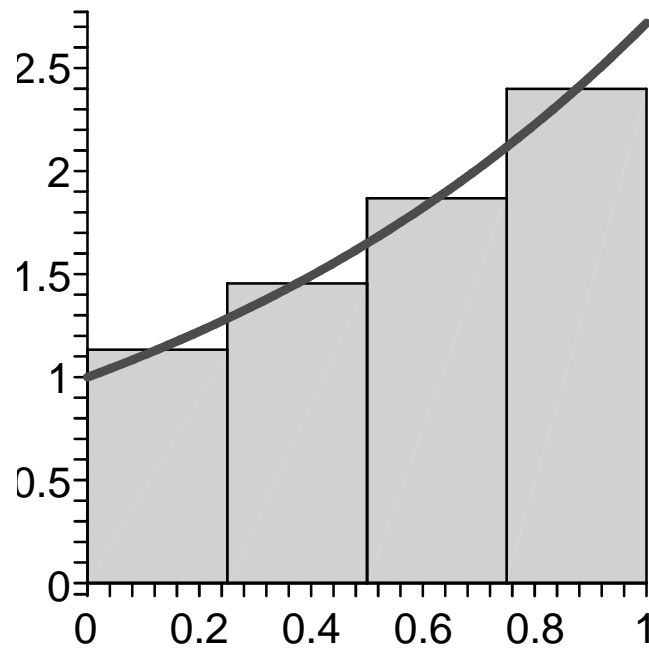
```
> middlebox(f(x),x=0..1,4);
```



```
> m4f:=evalf(sum(f(x4[i]+1/8)*Delta4,i=1..4));  
m4f:=0.2421875000
```

```
>
```

```
> middlebox(g(x),x=0..1,4);
```



```
> m4g:=evalf(sum(g(x4[i]+1/8)*Delta4,i=1..4));
      m4g := 1.713815280
```

```
> L4f,R4f,m4f;
      0.1406250000, 0.3906250000, 0.2421875000
```

```
> L4g,R4g,m4g;
      1.512436676, 1.942007133, 1.713815280
```

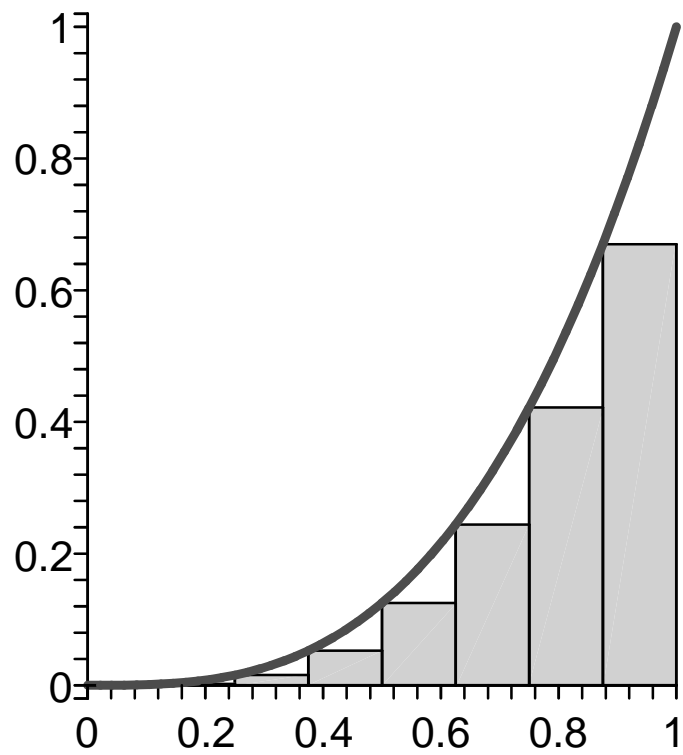
```
>
```

For n=8:

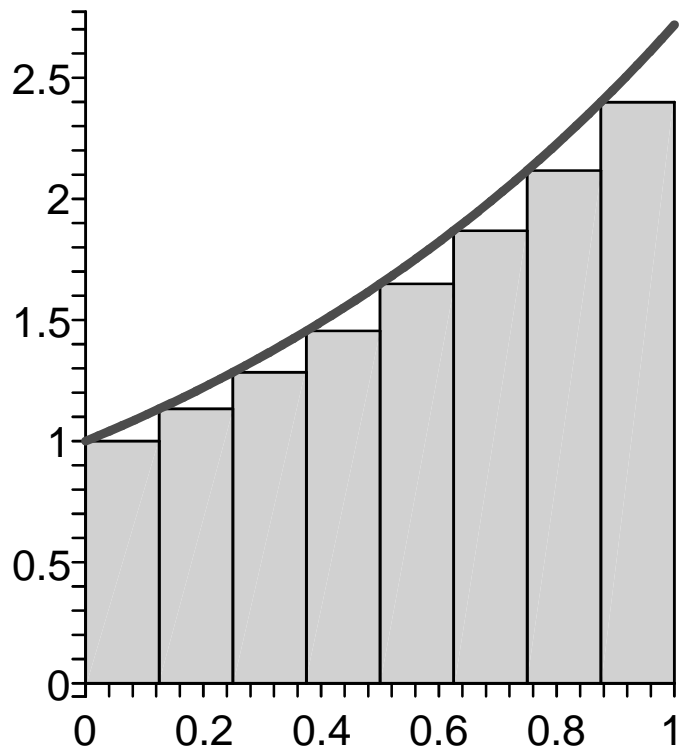
```
> x8:=[0,1/8,1/4,3/8,1/2,5/8,3/4,7/8,1];
      x8 := [ 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1 ]
```

```
> Delta8:=1/8;
      Δ8 := 1/8
```

```
> leftbox(f(x),x=0..1,8);
```

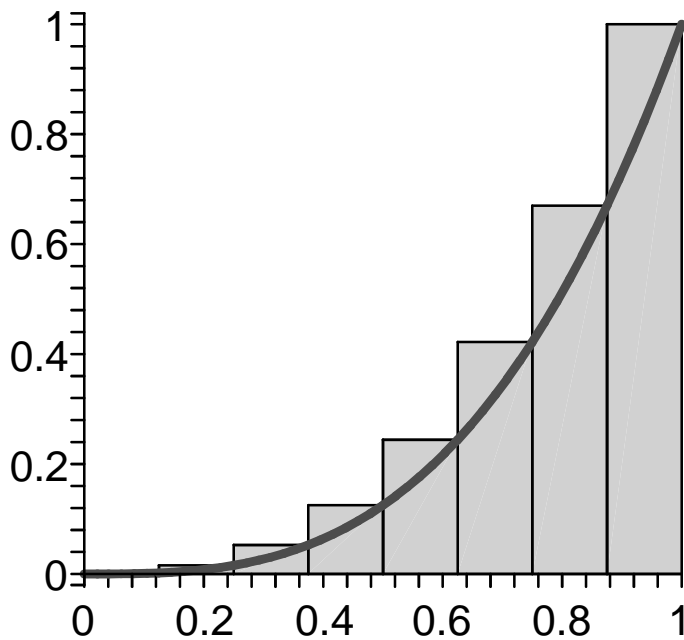


```
> L8f:=evalf(sum(f(x8[i])*Delta8,i=1..8));  
L8f:= 0.1914062500  
> leftbox(g(x),x=0..1,8);
```



```
> L8g:=evalf(sum(g(x8[i])*Delta8,i=1..8));
                L8g := 1.613125978
```

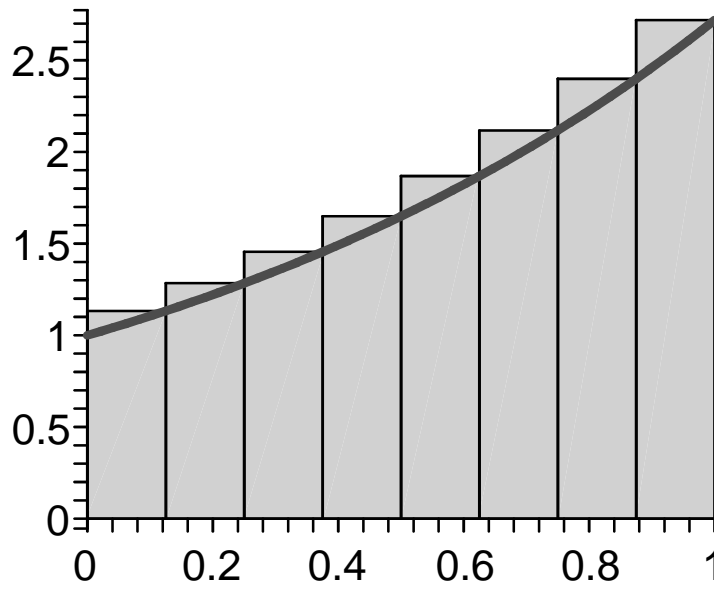
```
> rightbox(f(x),x=0..1,8);
```



```
> R8f:=evalf(sum(f(x8[i+1])*Delta8,i=1..8));
```

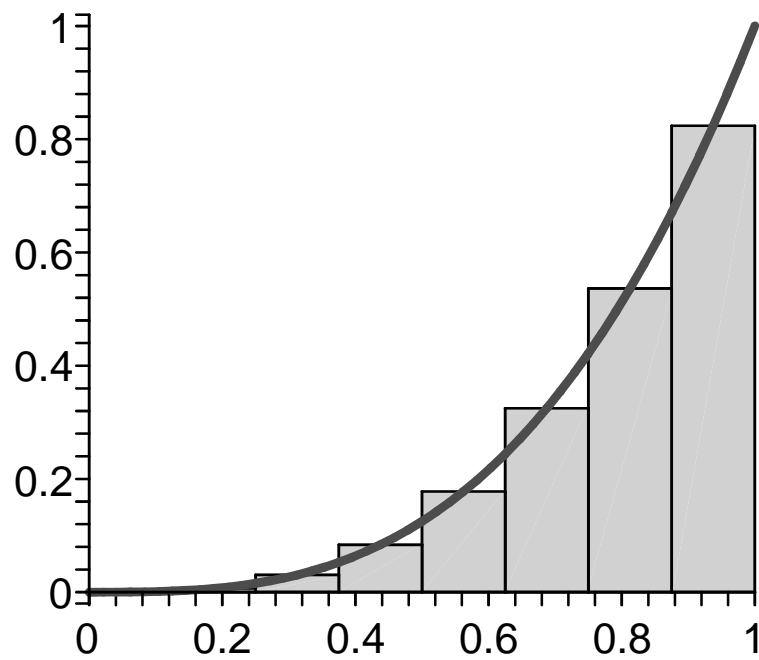
$R8f := 0.3164062500$

```
>  
> rightbox(g(x), x=0..1, 8);
```



```
> R8g:=evalf(sum(g(x8[i+1])*Delta8,i=1..8));  
R8g:=1.827911206
```

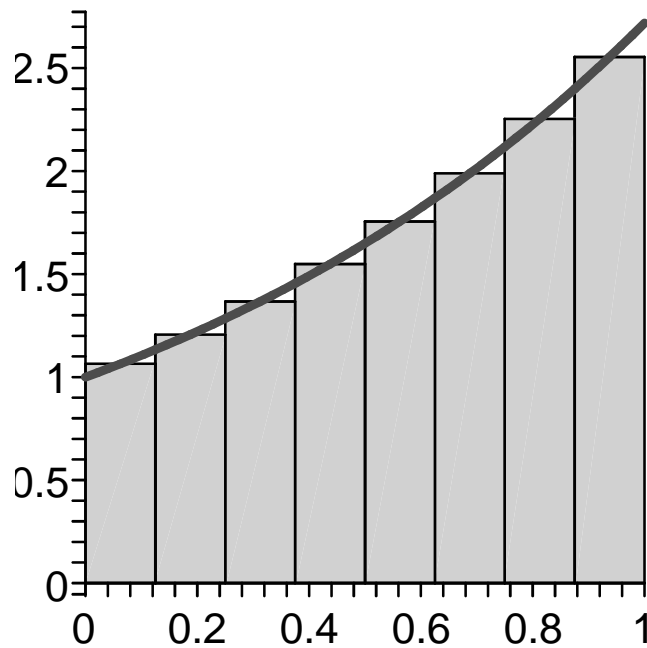
```
> middlebox(f(x), x=0..1, 8);
```



```
> m8f:=evalf(sum(f(x8[i]+1/16)*Delta8,i=1..8));
```


$m8f := 0.2480468750$

```
> middlebox(g(x), x=0..1, 8);
```



```
> m8g:=evalf(sum(g(x8[i]+1/16)*Delta8,i=1..8));  
m8g := 1.717163664
```

```
>
```

```
> L4f,R4f,m4f;  
0.1406250000, 0.3906250000, 0.2421875000
```

```
> L8f,R8f,m8f;  
0.1914062500, 0.3164062500, 0.2480468750
```

```
> abs(L4f-L8f), abs(R4f-R8f), abs(m4f-m8f);  
0.0507812500, 0.0742187500, 0.0058593750
```

```
> evalf(int(f(x), x=0..1));  
0.2500000000
```

```
> L4g,R4g,m4g;  
1.512436676, 1.942007133, 1.713815280
```

```
> L8g,R8g,m8g;  
1.613125978, 1.827911206, 1.717163664
```

```
> abs(L4g-L8g), abs(R4g-R8g), abs(m4g-m8g);  
0.100689302, 0.114095927, 0.003348384
```

```
> evalf(int(g(x), x=0..1));
```

1.718281828

[
[>
[>