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Lines

Problem: Describe lines in $\mathbb{R}^2$ and $\mathbb{R}^3$ systematically. Lines are determined by a point and a direction.

$\boxed{\mathbb{R}^2}$

- Familiar form: $y = mx + b$
- $(0, b)$ is a point on the line.
- m, the slope, determines the direction: $\vec{v} =< 1, m >$
- Rewrite:

$$< x, mx + b > = < 0, b > + x < 1, m >$$

General forms

- Vector form: Let the point be given by $\vec{r}_0$ and the direction be specified by a vector $\vec{v}$. Then the line is described by:

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

- Parametric form: If, in coordinates, $\vec{r}_0 =< x_0, y_0, z_0 >$, $\vec{v} =< a, b, c >$ and $\vec{r}(t) =< x(t), y(t), z(t) >$ then

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Symmetric form

Solving the parametric equations for t we have:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Examples:

- Find the equations of the line passing through the points $P = (1, 2, 3)$ and $Q = (-1, 4, 2)$.
- Does this line intersect the xy-plane? If so, where?
- Consider the lines $\vec{r}_1(t) =< t, t, t >$, $\vec{r}_2(t) =< 2 + t, 8 + t, t >$ and $\vec{r}_3(t) =< 1 + t, 10 - 2t, -1 + t >$. Which pairs of lines are parallel? Which cross? Which are orthogonal? Which are skew?