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Alernating Series

So far, we've (mostly) dealt with series that have only positive terms. What happens if there arre some negative terms. One common type of series with positive and negative terms are *alternating series*:

$$\sum_{n=k}^{\infty} (-1)^n a_n$$

where $\{a_n\}$ is a sequence of postive terms.

Alternating Series Test

If the alternating series

$$\sum_{n=k}^{\infty} (-1)^n a_n$$

satisfies

1. $a_{n+1} \geq a_n$ for all n
2. $$\lim_{n \to \infty} a_n = 0$$

Then the series converges.

Example:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

AST Examples

- $$\sum_{n=1}^{\infty} (-1)^n \frac{2n+4}{4n+1}$$

- $$\sum_{n=3}^{\infty} (-1)^n \frac{n}{2^n}$$

Absolute Convergence

When a series has both positive and negative terms, we say that a series

$$\sum_{n=k}^{\infty} a_n$$

converges absolutely if

$$\sum_{n=k}^{\infty} |a_n|$$

converges. If the series converges but does not converge absolutely, we say that it *converges conditionally*.

Examples

- $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$
  1. AST $\implies$ the series converges.
  2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the integral test.
  3. Conclusion: The series converges absolutely.
- $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$
  1. AST $\implies$ the series converges.
  2. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the integral test.
  3. Conclusion: The series converges conditionally.

Ratio test

Consider a series, $\sum_{n=k}^{\infty} a_n$. Then, if

1.
$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$$
then the series $\sum_{n=k}^{\infty} a_n$ converges absolutely.

2.
$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$$
or
$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$$
then the series $\sum_{n=k}^{\infty} a_n$ diverges.

Examples

- $$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$
- $$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$
- $$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$
- $$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$