m8w05, Quiz 1	Name:	Section:

Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

Problem 1: [8 pts] Find the following indefinite integral

$$\int \frac{9}{x^2\sqrt{9+4x^2}}dx$$

Solution:

Construct a right-triangle with hypotenuse $\sqrt{9+4x^2}$, opposite $2x$ and adjacent 3. This suggests the subsitution $\tan\theta = \frac{2x}{3}$. Then $dx = \frac{3}{2}\sec^2\theta d\theta$ and the integral becomes

$$\int \frac{4\sec\theta}{\tan^2\theta}d\theta = \int \frac{4\cos\theta}{\sin^2\theta}d\theta = -\frac{4}{\sin\theta} + C$$

Using the triangle, we see that $\sin\theta = \frac{2x}{\sqrt{9+4x^2}}$. And so our final solution is

$$-\frac{2}{x}\sqrt{9+4x^2} + C.$$

Problem 2: [9 pts] Find the area of the region bounded by the curves $y = \arctan x$ and $y = x\arctan x$.

Solution:

The two curves intersect when $\arctan x = x\arctan x$, which happens when $x = 1$ or $\arctan x = 0$, i.e at $x = 0, 1$. Between 0 and 1 we see $\arctan x \geq x\arctan x$. Thus the area is

$$Area = \int_0^1 (\arctan x - x\arctan x)dx.$$

Integrate by parts with $u = \arctan x$ and $dv = (1-x)dx$. Then $du = \frac{1}{1+x^2}dx$ and $v = x - \frac{1}{2}x^2$. Thus

$$\begin{aligned} Area &= \left[(x - \frac{1}{2}x^2)\arctan x\right]_0^1 - \int_0^1 \frac{x}{1+x^2} - \frac{1}{2}\frac{x^2}{1+x^2}dx \\ &= \frac{\pi}{8} - \int_0^1 \frac{x}{1+x^2} - \frac{1}{2} + \frac{1}{2}\frac{1}{1+x^2} \\ &= \frac{\pi}{8} - \frac{1}{2}\left[\ln|1+x^2| - x + \arctan x\right]_0^1 \\ &= \frac{\pi}{8} - \frac{1}{2}(\ln 2 - 1 + \frac{\pi}{4}) \\ &= \frac{1}{2} - \frac{1}{2}\ln 2 \end{aligned}$$

Problem 3: [8 pts] The unbounded region R is bounded above by the curve $y = \frac{1}{\sqrt{x^2+3x+2}}$, below by the $x - axis$ and to the left by $x = 0$. This region R is rotated about the $x - axis$. Is the volume of the resulting solid finite or infinite? If it is finite, evaluate it.

Solution:

The volume is given by the improper integral

$$Volume = \int_0^\infty \frac{\pi}{x^2+3x+2}dx = \lim_{t\to\infty} \int_0^t \frac{\pi}{(x+1)(x+2)}dx.$$

Apply partial fractions, i.e. find A and B so that $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$. Multiplying through yields $1 = A(x+2) + B(x+1)$. Choose $x = -1$ to see that $A = 1$ and $x = -2$ to see that $B = -1$. Then

$$\begin{aligned}
Volume &= \lim_{t\to\infty} \int_0^t \frac{\pi}{x+1} - \frac{\pi}{x+2}dx \\
&= \lim_{t\to\infty} \left[\pi \ln|x+1| - \pi \ln|x+2|\right]_0^t \\
&= \pi \lim_{t\to\infty} \ln\left|\frac{t+1}{t+2}\right| + \pi ln2 \\
&= \pi \ln 1 + \pi \ln 2 \\
&= \pi \ln 2.
\end{aligned}$$

Therefore the volume is finite and of value $\pi \ln 2$.