Math 8 Practice Exam 2 Problems

Disclaimer: This set of problems is meant neither to indicate the length nor composition of the actual exam. They are merely problems that we rejected for the actual exam. These problems are meant to help you practice for the exam. They are in no way a comprehensive set of problems - you may well see completely different types of problems on the exam.

1. Find the equations of the following lines and planes:
   (a) the plane through the points $(1,1,0)$, $(1,-4,2)$ and $(-2,3,1)$.
   (b) the plane through the origin and parallel to the plane $5x - 2y + z = 15$.
   (c) the line through the point $(1,2,3)$ and perpendicular to the plane $x - 3y + 5z = 1$.

2. (a) Find the area of the triangle with vertices $(1,-2,5)$, $(1,3,0)$ and $(1,0,1)$.
   (b) Find the volume of the parallelepiped determined by the vectors $\vec{\mathbf{u}} = \langle 1,2,3\rangle$, $\vec{\mathbf{v}} = \langle 2,0,1\rangle$ and $\vec{\mathbf{w}} = \langle 3,0,4\rangle$.

3. Consider the pair of planes
   $$x + y + 7z = 9 \text{ and } (2,-1,5) \bullet (x,y,z) = 6.$$
   (a) Find a vector representation of the line of intersection of these planes.
   (b) Find the cosine of the angle between these planes.

4. Consider the curve given parametrically by $g(t) = (t^2, \frac{1}{3}t^3),\ -2 \le t \le 2$.
   (a) Find the arc length of the curve from $t = 0$ to $t = 2$.
   (b) Find a vector representation of the tangent line to the curve at $t = 1$.
   (c) Suppose a particle is moving along the curve and then flies off at time $t = 1$ second, travelling at a constant speed along the tangent line. Find the position of the particle two seconds later.

5. (a) Find the projection of the vector $(1,3,-2)$ onto the vector $(1,1,1)$.
   (b) Write $(1,3,-2)$ as the sum of a vector parallel to $(1,1,1)$ and a vector perpendicular to $(1,1,1)$.

6. The curves $\vec{\mathbf{r}}_1(t) = \langle t^3, t^2, t\rangle$ and $\vec{\mathbf{r}}_2(t) = \langle \sin(t) + 1, e^t, \cos(t)\rangle$ intersect at the point $(1,1,1)$. Find the equation of the plane containing both tangent lines at $(1,1,1)$.

7. (a) Find the center and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 6x - 8z + 2 = 0$.
   (b) Find the arc length of the curve $\vec{\mathbf{r}}(t) = \langle t, \frac{t^3}{3} + 4, \frac{\sqrt{2}}{2}t^2\rangle$ from $t = 0$ to $t = 1$.

8. Are the lines $\frac{x-1}{4} = \frac{y-2}{-4} = \frac{z-3}{6}$ and $\vec{\mathbf{r}}(t) = \langle 3 + t, 2t, 6 + 3t\rangle$ skew, parallel, or intersecting?

9. A helicopter is to fly directly from the helipad at the origin toward the point $(1,1,1)$ at a speed of 60 feet/sec (and continue flying in that direction.)
   (a) Find a unit vector $\vec{\mathbf{u}}$ in the direction of motion.
   (b) Using part (a), what is the position of the helicopter after t seconds? after 10 seconds?

10. Consider the function $f(x,y) = \sqrt{1 - \ln(x^2 + y^2)}$
    (a) Find the domain of f.
    (b) Compute the limit as $(x,y) \to (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ of $f(x,y)$.
    (c) Compute the partial derivatives f_x and f_y.