Jury-rigging

M5 Crew

Wednesday February 25

Review

Monday we agreed that the SFMP, $E(XY) = E(X) \cdot E(Y)$ for bets based on independent events, implied that for A, B independent events we have $P(A \wedge B) = P(A) \cdot P(B)$.

Recall that $\wedge$ means "and."

Example

So, the probability of getting a 12 when rolling 2 dice is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ because you can think of rolling a 12 as rolling two consecutive 6's.

True Story

To see an example of how this has been (mis)used in real life, consider a case [in LA?] where the perpetrator was known to be an African-American in a particular age range who owned a certain model of car and dated a Caucasian women. I forget some details, but the argument went something like this:

Calculation

P(A given person in LA is an African-American)=$\frac{1}{4}$
P(A given person is in the proper age range)=$\frac{1}{5}$
P(A given person is dating someone of another ethnic group)=$\frac{1}{700}$
P(A given person owns a red car of a particular type)=$\frac{1}{1000}$

So...P(A person in LA matches all features)= $\frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{700} \cdot \frac{1}{1000} = \frac{1}{14000000}$

Highlander?

Pretending for the moment that the above are independent, let's see how likely it is that there are 2 people who match the description:

The likelihood that a given pair of two people will both math the description is $\frac{1}{14{,}000{,}000} \cdot \frac{1}{14{,}000{,}000}$ The total number of possible pairs is $\frac{4000000 \cdot 3999999}{2}$, or approximately 8 billion. Multiplying these numbers gives a probability of approx. $\frac{8}{196}$, a bit more than a 4 percent chance.

Verdict

Since the police found someone who matched all the characteristics, and there was only a 4 percent chance or so that there was another such person, he was convicted.

What they missed

What they originally missed in the trial [but was fixed on appeal] is that THERE WAS MORE INFORMATION. They were trying to calculate P(There are 2 people who match the description), but what they SHOULD have been doing was calculating P(There are 2 people who match the description GIVEN that we know there is at least 1)!

How to do it

This question is one of "Conditional Probability," that is the question of finding the probability of one thing GIVEN that you know something else is true. We can start by looking back at the first thing we mentioned today:

If A,B are independent events then $P(A \wedge B) = P(A) \cdot P(B)$.

Now, to think about situations where A and B may not be independent, think of $A \wedge B$ as being two events that happen in a particular order [actually, the order does not matter, but it helps for thinking about it].

$P(A \wedge B) = P(A) \cdot$ P(B given that A occurs).

Big Rule 1

Which let's us say:
P(B given that A occurs)=$\frac{P(A \wedge B)}{P(A)}$

This is very useful. We write "B happens given that A happened" as "$B|A$"

Big Rule 2

In this case if we let B be "There are at least two such men" and let A be "There is at least one such man." Then we have:
$P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{P(B)}{P(A)}$

In the LA case, it means that the likelihood of having two men GIVEN THAT THERE IS AT LEAST 1 is

$\frac{\frac{4}{100}}{\frac{2}{7}}$ Or 14 percent...enough to get the appellate court to overturn the original conviction. [Note, I said that the likelihood that there is at least 1 was 2/7, this is a bit of an overestimate, you can calculate this like we did the birthday problem earlier].