Math 3 Winter 2003
Practice Problems

This is NOT a "practice final;" neither the length nor the composition represents an actual final exam. These are practice problems that use the material we studied after the second midterm (and that may also help to review some earlier material.)

1. Sketch the graph of

$$f(x) = x - 2\tan^{-1}(x),$$

   paying attention to:

   symmetry

   asymptotes

   x- and y-intercepts

   local maxima and minima

   inflection points

   behavior as $x \to \infty$ and $x \to -\infty$

2. Let $f(x) = \ln(x)$.

   (a) Use the limit definition of derivative to write down a limit that equals $f'(1)$.

   (b) Use the fact that you know how to differentiate f to write down a number that equals $f'(1)$.

   (c) Use the above to find

   $$\lim_{h \to 0} \ln\left((1+h)^{\frac{1}{h}}\right).$$

   $$\lim_{h \to 0} \left((1+h)^{\frac{1}{h}}\right).$$

3. (a) Find:

   $$\frac{d}{dx}\left(x\ln(x) - x\right);$$

   $$\int_1^2 \ln(x)\, dx.$$

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(b) Find
$$\int xe^{(x^2)}\, dx.$$

(c) Find
$$\lim_{x\to 0^-} e^{\left(\frac{1}{x}\right)}$$

4. The area under the curve
$$y = \frac{1}{\sqrt{4x^2 + 1}}$$
for $0 \le x \le 4$ is revolved around the x-axis. Find the resulting volume.

5. Suppose that $f(x)$ is some unknown function, and you know that f is one-to-one (so it has an inverse function), $f(2) = 3$ and $f'(2) = 5$. If
$$g(x) = f^{-1}(x^2 - 1),$$
find $g'(2)$.

6. The decibel level of a sound is defined in terms of its intensity; if the intensity of the sound is I (measured in watts per square meter), the decibel level is
$$L = 10 \log_{10}\left(\frac{I}{I_0}\right),$$
where I_O is a constant and the units of L are decibels (dB.)

The value of I_0 is 10^{-12} (a sound having frequency 1000 hertz and intensity 10^{-12} watts per square meter is barely audible.)

At one point while the amplifier is being cranked up, the decibel level in your room is measured to be 100 dB and is increasing at a rate of 2 dB per second. How fast is the sound intensity increasing?

(Your answer should be a number with units. It is okay if your number is not completely simplified, expressions like $\ln(3)$ or 3π are fine, but please do the easy simplifications; an expression like $\sin(\pi)$ or 3^2 should be simplified.)

7. A triangle has one vertex A at the point $(1, 0)$ on the x-axis, one vertex B at the point $(2, 0)$ on the x-axis, and its third vertex C at a point $(y, 0)$ on the y-axis. What value of y will give the largest possible value to the angle θ at the vertex C?