Math 3 Winter 2003
Practice Problems for Exam II

1. Short answer problems.

   (a) A certain mushroom changes shape as it grows, so its weight W (measured in grams) is a complicated function of its height h (measured in centimeters.) If you know that

   $$\int_3^5 \frac{dW}{dh}\, dh = 2,$$

   then what can you conclude?
       i. An average mushroom between 3 and 5 centimeters tall weights 2 grams.
       ii. A 5 centmeter tall mushroom weighs 2 grams more than a 3 centimeter tall mushroom.
       iii. A 5 centimeter tall mushroom weighs twice as much as a 3 centimeter tall mushroom.
       iv. A mushrooom between 3 and 5 centimeters in height that grows a small amount Δh centimeters taller will gain about $2\Delta h$ grams in weight.

   (b)
   $$F(x) = \int_{-x}^{x} \cos^2(t)\, dt.$$

   What is the derivative $F'(x)$?

   (c) TRUE or FALSE?
       i. On any interval on which the function f is differentiable,

       $$\int f'(x)\, dx = f(x) + C.$$

       ii. The antiderivative of the velocity function of a moving particle gives you the particle's acceleration.
       iii. If $f(x) = f(-x)$ for all x, and f is continuous everywhere, then for all a,
       $$\int_{-a}^{a} f(x)\, dx = 0.$$

1

2. (a) The sum
$$1^2 \Delta x + 3^2 \Delta x + 5^2 \Delta x + 7^2 \Delta x + 9^2 \Delta x$$

is the fifth left-endpoint Riemann sum (L_5) approximating
$$\int_a^b x^2 \, dx.$$

Provide the following numbers:
$$a =$$
$$b =$$
$$\Delta x =$$

(b) If f is continuous and
$$\int_0^{\sqrt{3}} f(x) \, dx = -1,$$
$$\int_0^3 f(x) \, dx = 1,$$
$$\int_0^9 f(x) \, dx = 4,$$

find
$$\int_0^3 x f(x^2) \, dx.$$

(You may not need to use all the given information.)

(c) What is
$$\int \frac{1}{\cos^2(x)} \, dx?$$

3. (a) Evaluate
$$\int_0^\pi \sin^3 x \, dx.$$

(b) Evaluate
$$\int_{-1}^1 (x+2)\sqrt{1-x^2} \, dx$$

by writing it as a sum of integrals and interpreting one of them as an area.

4. A solid object has a base that is a right triangle. One leg has length $2''$ and the other has length $4''$. When it is sliced vertically with slices parallel to the $2''$ leg of the base, the cross-sections are half-discs. Find its volume.

5. A covered aquarium has a rectangular base, and must be twice as tall as the width of the base. If material for the top of the aquarium costs $5 per square foot, material for the bottom costs $8 per square foot, and glass for the sides costs $2 per square foot, find the volume of the largest aquarium that can be built for $100 or less.