Math 3 Winter 2003
Practice for Exam I

1. (a) i. Complete the statement:
The Intermediate Value Theorem tells us that if f is continuous on $[a, b]$, and

$$f(a) = A < C < B = f(b),$$

then:

ii. Complete the statement:
If f is a continuous function on $(-\infty, \infty)$, $f'(x) < 0$ for every $x < 0$, and $f'(x) > 0$ for every $x > 0$, then at the point $x = 0$, f must have:

(b) TRUE OR FALSE

i. It is possible to have a function $f(x)$ that is continuous and differentiable on $(-\infty, \infty)$, such that $f'(x) \geq 1$ for every x, and $\lim_{x\to\infty} f(x) = 0$.

ii. It is possible to have a continuous function on $(0, 1)$ that has a maximum value on $(0, 1)$.

iii. Since $\sin 0 = 0$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, the Mean Value Theorem guarantees us that for some c between 0 and $\frac{\pi}{6}$,

$$\cos c = \frac{3}{\pi}.$$

iv. $\lim_{h\to 0} \frac{tan\left(\frac{\pi}{12} + h\right) - tan\left(\frac{\pi}{12}\right)}{h} = \sec^2\left(\frac{\pi}{12}\right).$

2. (a) Find the slope of the line tangent to $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

   (b) The volume of a cone is given by $V = \dfrac{\pi r^2 h}{3}$, where r is the radius and h is the height.

      i. Find the rate of change of the volume with respect to height if the radius is constant and equals 3 cm.

      ii. find the rate of change of the volume with respect to radius if the height is constant and equals 10 cm.

3. An amount of gas having mass 1 gram is enclosed in a spherical balloon of radius r. As the temperature rises, the gas expands, so r increases.

   The volume of gas

   $$V = \frac{4}{3}\pi r^3$$

   also increases.

   The density of the gas

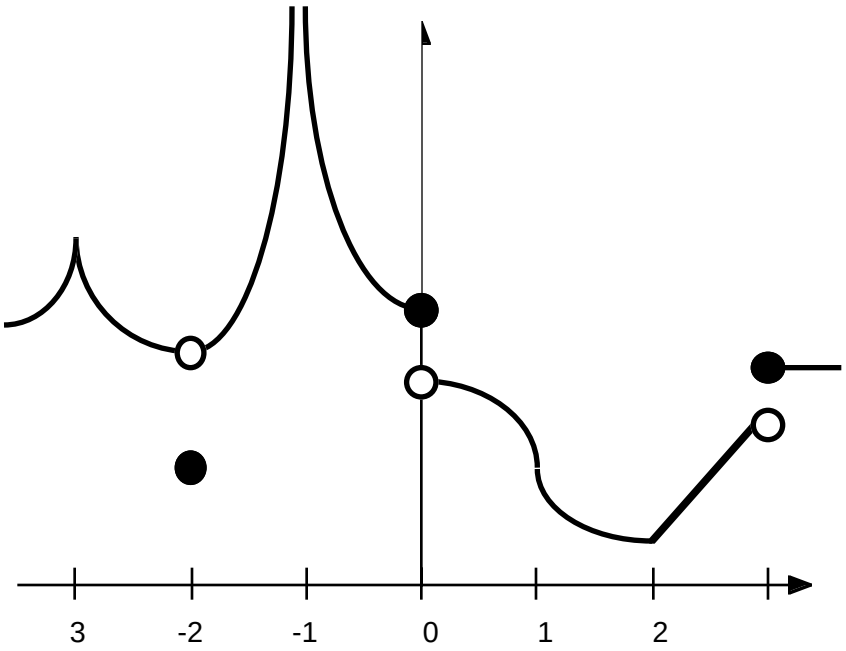
   $$D = \frac{mass}{volume}$$

   decreases.

   When the radius r of the balloon is 1 meter, r is increasing at a rate of .1 meters per second.

   How fast is V increasing?

   How fast is D decreasing?



4. (a) Here is the graph of a function $f(x)$.

   i. At which values of x is f discontinuous?

   ii. At which values of x does f have removable discontinuities?

   iii. At which values of x does f have vertical asymptotes?

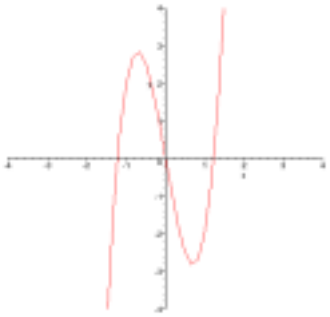
   iv. At which values of x is f *not* differentiable?

   v. At which values of x does the derivative of f, f', have removable discontinuities?

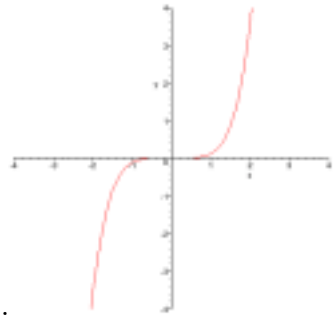
   vi. At which values of x does the derivative of f, f', have vertical asymptotes?

(b) Picture (A.) is the graph of a second derivative $f''(x)$. One of the other pictures is the graph of f. Which one? How can you tell?

A.

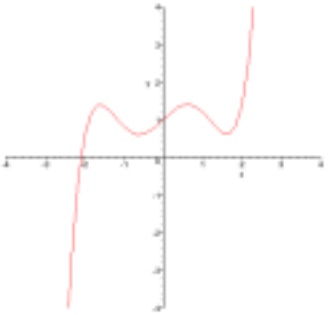


B.



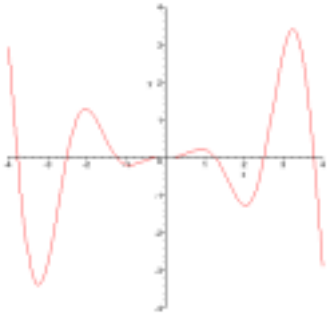
C.

D.



E.

F.



5.

$$f(x) = \sqrt{\frac{x-1}{x+1}}$$

(a) Where is $f(x)$ undefined? (Don't forget the square root of a negative number is undefined.)

(b) Find:

$$\lim_{x \to (-1)^-} f(x)$$

$$\lim_{x \to (1)^+} f(x)$$

$$\lim_{x \to \infty} f(x)$$

$$\lim_{x \to -\infty} f(x)$$

(c) Find the derivative $f'(x)$

(d) Find

$$\lim_{x \to (1)^+} f'(x)$$

(e) Sketch the graph of f, taking all this into account.