

**HOMEWORK 1**  
**DUE JANUARY 17, 2006**

- (1) Show that if  $\mathbb{F} = \mathbb{C}$ , a sesquilinear form is self-adjoint if and only if  $\langle x, x \rangle \in \mathbb{R}$  for every  $x \in \mathcal{H}$ .
- (2) Suppose that  $\langle \cdot, \cdot \rangle$  is an inner product on the vector space  $\mathcal{H}$ . Prove the following *polarization identity*:

(1) 
$$4\langle x, y \rangle = \sum_{k=0}^3 i^k \|x + i^k y\|^2;$$

Show that if a norm satisfies the parallelogram law, then the equation 1 will define an inner product on  $\mathcal{H}$ .

- (3) Problems #3,6,11 from pages 6–7.
- (4) Problems # 2,3,5 from page 11. For problem 5 you can use (without proof) problem 4.
- (5) Problem #2, 4 from page 13.