# On the Nature of Gödel's Second Incompleteness Theorem

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Gödel's "Second" Incompleteness Theorem states that axiom systems of sufficiently great strength are unable to formally verify their own consistency. Let $A(x, y, z)$ denote a 3-way predicate relation indicating that $x + y = z$, and let $M(x, y, z)$ indicate that $x * y = z$. Let us say an axiom system $\alpha$ *recognizes* addition and multiplication as **"Total"** functions iff it can prove:

$$\forall x \; \forall y \; \exists z \;\; A(x,y,z) \quad \text{AND} \quad \forall x \; \forall y \; \exists z \;\; M(x,y,z) \; . \tag{1}$$

In several recent articles, we have shown how such totality conditions are related to both generalizations and boundary-case style exceptions for Gödel's Second Incompleteness Theorem. This talk will survey several of our most recently published results [1, 2, 3, 4, 5, 6, 7] about this subject.

## References

[1] D. Willard, "Self-Verifying Systems, the Incompleteness Theorem and the Tangibility Reflection Principle", *Journal of Symbolic Logic* 66 #2 (2001) pp. 536-596

[2] D. Willard, "How to Extend The Semantic Tableaux And Cut-Free Versions of the Second Incompleteness Theorem Almost to Robinson's Arithmetic Q", *Journal of Symbolic Logic* 67 #1 (2002) pp. 465-496.

[3] D. Willard, "A Version of the Second Incompleteness Theorem For Axiom Systems that Recognize Addition But Not Multiplication as a Total Function", *First Order Logic Revisited,* Logos Verlag (Berlin) 2004, pp. 337–368.

[4] D. Willard, "An Exploration of the Partial Respects in which an Axiom System Recognizing Solely Addition as a Total Function Can Verify Its Own Consistency", *Journal of Symbolic Logic* 70 #4 (2005) pp. 1171-1209.

[5] D. Willard, "On the Partial Respects in which a Real Valued Arithmetic System Can Verify its Tableaux Consistency". Conference version of this result in SpringerVerlag LNCS#3702, (2005), pp. 292-306, with a more polished and extended journal-style version of this result to appear in the December 2006 issue of the *Journal of Symbolic Logic*.

[6] D. Willard, "A New Variant of Hilbert Styled Generalization of the Second Incompleteness Theorem and Some Exceptions to It", *Annals of Pure and Applied Logic* 141 (2006) pp. 472-496.

[7] D. Willard, "The Axiom System $I\Sigma_0$ Manages to Simultaneously Obey and Evade the Herbrandized Version of the Second Incompleteness Theorem", to appear in 2006 in *Electronic Notes in Theoretical Computer Science*, (in the journal's volume containing the Proceedings of the Wollic-2006 Conference).