CATEGORIZATION OF PURE SHUFFLES

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Given a finite set of linear orderings $S = S_i$, we define their shuffle, $\sigma(S)$ in such a way that $\sigma(S) + S_i + \sigma(S)$ is isomorphic to $\sigma(S)$, for each S_i in S. The shuffle operator plays an important role in classifying the set of linear orderings and the set of countably categorical linear orderings. We will be examining a subset of these, the pure shuffles, which can be obtained from the set of finite linear orderings by finitely many shuffle operations. Examining these constructions, we find equivalences such as $\sigma(\sigma(1)) = \sigma(1)$, and $\sigma(\sigma(1, 2), 2) = \sigma(1, 2) = \sigma(2, 1)$, and may wonder whether all of these equivalences can be given a combinatorial classification. It turns out that there is a simple classification of these equivalences, which I will present, representing work done between myself and Francois Dorais. If time permits, I will discuss applications of this categorization to the problem of counting logical equivalence classes on linear orderings.