Infinite Hat Problems

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July 25, 2007

Suppose that there are infinitely many people in a room. Colored hats are placed on their heads, such that everyone can see the hats of the other players but not their own, and they must attempt to guess the colors of their own hats. How well can they do? Yuval Gabay and Michael O'Connor demonstrated a finite-error strategy (a strategy guaranteeing that only finitely many players guess wrong). More generally, one can put an arbitrary binary relation on the set of players that specifies which players can see which hats, leading to a natural question: For which of these binary relations does a finite-error strategy exist? We will present an answer to this question in the case of countably many players, and a partial solution in the case of uncountably many players. We will also look at a closely related result: there is a strategy which, given an arbitrary function f from the reals to the reals, attempts to guess f(t) from the values of f(x), x ¡ t, and these guesses are only wrong countably often. This raises interesting questions regarding the philosophical problem of induction. (Joint work with Alan Taylor of Union College.)