Math 60 pre-retest

1. An urn contains 20 indistinguishable balls. In five draws you remove all the balls, without replacement. For example, you might draw $0, 20, 0, 0, 0$, or $5, 5, 4, 4, 2$, etc.

   (a) What is the probability of drawing all the balls in one draw?

   (b) What is the probability you draw at least one ball per draw?

2. A bus has 5 stops and at the beginning of its trip it has 20 passengers. If passengers can get off at each stop and no passengers get on, what is the probability that the bus is empty by the last stop (stop 5)?

3. How many license plates are there satisfying simultaneously all of the following conditions:

   (a) 3 digits (0–9), 2 letters

   (b) first spot is a letter

   (c) the number formed is divisible by 6

   For example: $A4Z50$ is valid because $6|450$.

4. Show that
$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}.$$

5. A senate has 100 members, 2 from each state in the union. A committee is formed of 50 senators. What is the probability that the committee will have a member from each state?

6. There are 20 people at Dartmouth. The people are going to be split into 2 groups of 10 people. What is the chance that any two people are going to be in the same group?

7. There are a total of 600 bears, 30 of which are defective. If you buy 30 bears, what is the probability that exactly 15 are defective?

8. There are 100 distinguishable boards to be sanded and painted. In how many orders can we sand and paint all the boards if each board needs to be sanded before it is painted (a board need not be painted immediately after it is sanded)?

9. How many ways can three committees of five people be chosen from a population of twenty people with two committee leaders when people can only be members of one committee? Two committees? All three?

10. A house with three distinguishable rooms will be used to imprison twelve indistinguishable trolls. If no room may be empty, how many ways may we imprison these trolls?

11. Starting at one vertex of a cube, take a random walk of 3 steps (backsteps are possible). What is the probability you have arrived at the opposite vertex from the start?