

The largest prime factor of a Mersenne number

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Abstract

A Mersenne number is a number of the form $M_n = 2^n - 1$. If it is prime, it is called a Mersenne prime. Euclid knew that each Mersenne prime gives an even perfect number and Euler showed that this correspondence goes the other way too. The Mersenne numbers are easy to test for primality thanks to a test due to E. Lucas. Although it is suspected that there are infinitely many Mersenne primes, this has not been proved yet. There are 44 known Mersenne primes and the largest one is $M_{32,582,657}$. It has 9,808,358 digits. Discovered on September 4, 2006, it is the largest number that has been proved prime.

In my talk, I will survey various known arithmetical properties of Mersenne numbers with an emphasis on lower bounds for the largest prime factor $P(M_n)$ of M_n . I will also report on the fact that the series $\sum 1/P(M(n))$ is convergent, a result obtained recently in joint work with Kevin Ford and Igor Shparlinski.