

A bijection between 2-triangulations and pairs of non-crossing Dyck paths

Sergi Elizalde
Dartmouth College

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(Note unusual time and place!)

Abstract

Triangulations of a convex polygon are known to be counted by the Catalan numbers. A natural generalization of a triangulation is a k -triangulation, which is defined to be a maximal set of diagonals so that no $k+1$ of them mutually cross in their interiors. It was proved by Jakob Jonsson that k -triangulations are enumerated by certain determinants of Catalan numbers, that are also known to count k -tuples of non-crossing Dyck paths.

There are several simple bijections between triangulations of a convex n -gon and Dyck paths. However, no bijective proof of Jonsson's result is known for general k . In this talk I will give a bijective proof for the case $k = 2$, that is, I will present a bijection between 2-triangulations of a convex n -gon and pairs (P, Q) of Dyck paths of semilength $n - 4$ so that P never goes below Q . The bijection is obtained by constructing isomorphic generating trees for the sets of 2-triangulations and pairs of non-crossing Dyck paths.