Decomposition of Fronts and Their Role in Contact Geometry

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(Tea 3:30 pm Math Lounge)

Abstract

A front in the plane is a closed smooth curve which is allowed to have isolated cusps and transverse self-intersections, but is not allowed to have vertical tangents. Fronts arise as xz-projections of "Legendrian knots", that is closed non-self-intersecting Legendrian curves in xyz-space; a curve $x = x(t)$, $y = y(t)$, $z = z(t)$ is called Legendrian if it satisfies the equation $x'y = z'$. (The true origin of this equation lies in classical mechanics, but this connection will not be exploited in the talk.)

Works of several last years demonstrated a crucial importance of a certain geometric property of fronts: the existence of a "decomposition" aka a normal ruling (introduced independently by Chekanov-Pushkar and the speaker). For a front with $2n$ cusps (n left cusps and n right cusps) it is a presentation of the front as the union of n non-self-intersecting closed curves, each contains one left cusp and one right cusp and intersects any vertical line between them in exactly two points. A certain "normality condition" (its description requires a short paragraph, I prefer to save it for the talk) is also imposed.

The existence of a decomposition turns out to be a necessary and sufficient condition for at least three, seemingly unrelated to each other, properties of a Legendrian knot: one comes from Analysis, one from Algebra, and one from Topology.

Although the talk will be related to many mathematical disciplines, no preliminary familiarity with any of them is required or expected.

This talk should be accessible to undergraduates.