Rational generating functions and compositions

Bruce Sagan
Michigan State University

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(Tea 3:30 pm Math Lounge)

Abstract

A *composition* of the nonnegative interger n is a way of writing n as an ordered sum of positive integers. So the compositons of 3 are $1+1+1$, $1+2$, $2+1$, and 3 itself. It is well-known and easy to prove that if c_n is the number of compositions of n, then

$$c_n = \begin{cases} 1 & \text{if } n = 0 \text{, and} \\ 2^{n-1} & \text{if } n \geq 1. \end{cases}$$

Equivalently, letting x be a variable, we have the generating function (power series)

$$\sum_{n \geq 0} c_n x^n = \frac{1-x}{1-2x}$$

which is a rational function. We show that this is a special case of a more general family of rational functions associated with compositions. Our techniques include the theory of formal languages from computer science. Surprisingly, identities about hypergeometric series are needed to do some of the computations.

This talk should be accessible to graduate students.