Extremal quasiconformal mappings

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007 Kemeny Hall, 4:00 pm
(Tea 3:30 pm 300 Kemeny Hall)

Abstract

Quasiconformal mappings are an extension of the class of conformal transformations, designed to be flexibile enough to avoid rigidity, while at the same time still share some aspects of conformality. Roughly speaking, while a conformal transformation sends infinitesimal spheres into spheres, a quasiconformal mapping will send infinitesimal spheres into ellipsoids with a uniform bound on the ratio of the axis. The maximum ratio is called 'dilation'. Starting from the work of Grotsch, Teichmueller, Ahlfors, Lavrentieff in the 1930's, such mappings have played a key role in the solution of several problems within mathematics (e.g. in complex analysis, PDE, inverse problems) and in applications to other sciences. The study of quasiconformal mappings rests on a rich and lively mix of techniques and ideas from analysis, geometric measure theory, geometry, topology, complex analysis and PDE.

Extremal problems for quasiconformal mappings have first appeared in 1928 in the work of Grotsch. Such problems typically involve finding a quasiconformal mapping between two spaces with minimal dilation (i.e. closest to conformal) in a given class of competitors (obtained for instance by fixing boundary data or a specific homotopy class). Extremal quasiconformal mappings are at at the basis of Teichmueller theory and continue to be an interesting topic.

In this talk I will give a very brief outline of the history of the extremal problem and mention a recent (PDE based) approach (joint with A. Raich) aimed at obtaining a qualitative study of minimizers. This approach is based on a set of techniques, originally developed by Aronsson in the 60's to study minimal Lipschitz extensions,

The talk will be aimed at a general audience, no prior knowledge of quasiconformal mappings or of extremal problems will be assumed.

This talk should be accessible to graduate students.

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