Random Walks on Groups, Switching Systems and Limiting Distributions

Clyde Martin
Texas Tech University

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007 Kemeny Hall, 4:00 pm
(Tea 3:30 pm 300 Kemeny Hall)

Abstract

We have extensively studied the systems

$$\dot{x} = (\delta(t)A + (1-\delta(t))B)x(t), \quad \delta(t) \in \{0,1\},$$

$$\begin{aligned} dx &= (z(t)A + (1-z(t))B)dt \\ dz &= (1-2z)dN_\lambda \end{aligned}$$

and

$$x_{n+1} = (\delta_1 A_1 + \cdots + \delta_k A_k)x_n$$

where $\delta_i \in \{0,1\}$, $\delta_1 + \cdots + \delta_k = 1$ and $P(\delta_i = 1) = p_i$ and we have a fairly good understanding of when these systems are stable. Our interest has now moved to an attempt to understand what happens when they are not stable. Interesting phenomena occurs when we consider a representation of a finite group G, i.e. a homomorphism, L, from G into $Hom(V,V)$.We now consider the system

$$X_{n+1} = (\delta_1 L(g_1) + \cdots + \delta_k L(g_k))xX_n$$

where $\delta_i \in \{0,1\}$, $\delta_1 + \cdots + \delta_k = 1$, $P(\delta_i = 1) = p_i$ and $X_0 = I$. Because $X_n = L(g)$ for some $g \in G$ the system evolves on the image of the representation. Using linear system theory we can calculate all of the moments of the limiting distribution and using the fact that the system creates a Markov process we can calculate the transition probability matrix. A nice interplay between systems theory and the work of Perci Diaconis on the role of groups in probability and statistics develops. The results have applications in such diverse areas as magic and genetics.

This talk should be accessible to graduate students.

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