Quantization and Reduction

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Abstract

In this talk, I will discuss two important aspects of modern symplectic geometry: geometric quantization and symplectic reduction. Symplectic geometry is historically rooted in Hamiltonian mechanics. The problem of quantization, that is, how to pass from Hamiltonian mechanics to quantum mechanics, starting with Dirac in the 1920s, has become a fundamental problem in mathematical physics. Geometric quantization is one prescription that is mathematically very interesting, lying at the intersection of symplectic geometry, representation theory and functional analysis. Another classical problem in mathematical physics is: how does one account for symmetry? In mathematics, a quotient space usually describes what is left after symmetry is removed from a system, but when dealing with Hamiltonian mechanics (i.e., symplectic manifolds) the situation is more complicated, and the correct notion of "quotient" is given by symplectic reduction. We will consider how these two problems intersect: how does one account for symmetry in geometric quantization. A result of Guillemin and Sternberg (1985), commonly referred to as "quantization commutes with reduction", states that there is a certain isomorphism between "reduce then quantize" and "quantize then reduce". In this talk (which will be accessible to non-specialists), we will review the basic facts of the case, and hopefully see that the situation is actually a bit more subtle.

This talk should be accessible to graduate students.