# The index of an algebraic variety 

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#### Abstract

Let $K$ be a field. Suppose that the algebraic variety is given by the set of common solutions to a system of polynomials in $n$ variables with coefficients in $K$. Given a solution $P=\left(a_{1}, \ldots, a_{n}\right)$ of this system with coordinates in the algebraic closure of $K$, we associate to it an integer called the degree of $P$, and defined to be the degree of the extension $K\left(a_{1}, \ldots, a_{n}\right)$ over $K$. When all coordinates $a_{i}$ belong to $K, P$ is called a $K$-rational point, and its degree is 1 . The index of the variety is the greatest common divisor of all possible degrees of points on $P$. It is clear that if there exists a $K$-rational point on the variety, then the index equals 1 . The converse is not true in general. We shall discuss in this talk various properties of the index.


