The index of an algebraic variety

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Abstract

Let K be a field. Suppose that the algebraic variety is given by the set of common solutions to a system of polynomials in n variables with coefficients in K. Given a solution $P = (a_1, \ldots, a_n)$ of this system with coordinates in the algebraic closure of K, we associate to it an integer called the degree of P, and defined to be the degree of the extension $K(a_1, \ldots, a_n)$ over K. When all coordinates a_i belong to K, P is called a K-rational point, and its degree is 1. The index of the variety is the greatest common divisor of all possible degrees of points on P. It is clear that if there exists a K-rational point on the variety, then the index equals 1. The converse is not true in general. We shall discuss in this talk various properties of the index.

This talk should be accessible to graduate students.