

Do symmetric groups have tilting modules?

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007 Kemeny Hall, 4:00 pm
(Tea 3:30 pm 300 Kemeny Hall)

Abstract

Representation theory combines elements of group theory and linear algebra. We begin with a gentle introduction to the representation theory of finite groups. The subject depends crucially on two variables: which groups are being studied and what field the vector spaces are defined over.

We will give examples that illustrate the fundamental difference between the ordinary (fields of characteristic zero) theory and the modular (characteristic p) theory.

Next we discuss some representation theory of the symmetric group and the general linear group, and describe how one can pass information back and forth between the two settings. We discuss modules with Specht filtrations for the symmetric group. Until recently it was believed no nice theory of such modules could exist, since well-known examples in characteristic two and three demonstrated that filtration multiplicities are not even well-defined. In 2004 the speaker and Nakano proved the surprising result that this bad behavior occurs only in these small characteristics, opening up the possibility of developing a general theory.

We will discuss this result and then present some more recent work on developing a theory for the symmetric group partially analogous to the theory of good filtrations for algebraic groups, including a conjectural description of the symmetric group analogue of tilting modules and a recent result about irreducible Specht modules that provides evidence for this conjecture. Along the way we will encounter many of the important open problems in the field.

The talk should be accessible to beginning graduate students and undergraduates who have taken, for example, Math 22 and 31.