# An elementary proof of a theorem of Douglas and Foias on invariant operator ranges

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**Abstract**

All of the main structure theorems for $n \times n$ complex matrices (e.g., upper triangular form, Jordan form) can be expressed solely in terms of invariant spaces. For bounded operators on an infinite-dimensional space it is not known whether every operator even has a nontrivial closed invariant subspace. However, C. Foias showed that if closed linear subspaces are replaced with operator ranges (i.e., ranges of operators), most of the finite-dimensional results (e.g., Burnside's theorem) are true in infinite dimensions. Douglas and Foias proved a striking theorem: If $T$ is a non-algebraic operator and $S$ is an operator that leaves invariant every $T$-invariant operator range, then there is an entire function $f(z)$ such that $S = f(T)$. We give a proof that mostly uses very elementary linear algebra.

This talk should be accessible to bright undergraduates who have had a good course in linear algebra.

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